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WHAT DO WE ESTIMATE IN PRODUCTION FUNCTION REGRESSIONS? CRITIQUE AND NEW APPROACHES

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Abstract

This paper demonstrates that, under weak assumptions, estimates from production function regressions using firm-level data are often inconsistent with profit maximization or imply implausibly large profits. Theoretically, the puzzle can be reconciled by relaxing the assumption of the perfect elasticity of factor supplies. Econometrically, the puzzle arises because of the transmission bias in OLS, endogeneity of factor prices in FIML/IV, and poor identification inversion-based/control-function estimators and GMM/IV estimators that use lags of endogenous variables as instruments. I argue that simple structural estimators can address both theoretical and econometric problems. Specifically, the paper proposes a full-information estimator that estimates the cost and the revenue functions simultaneously and treats unobserved heterogeneity in productivity and factor prices symmetrically. The strength of the proposed estimator is illustrated by Monte Carlo simulations and an empirical application. Finally, the paper argues that the profit share in revenue is a robust non-parametric *economic* diagnostic for estimates of returns to scale.

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1. INTRODUCTION

Production functions estimated on firm-level data sets can provide important insights into micro- and macroeconomic phenomena. The micro-level data, however, have important limitations. At the firm level, revenue and costs are well measured but prices and quantities are not. Commonly, firm revenues are deflated by industry price indices to get a measure of quantity. I argue that this measure of quantity, when used as the dependent variable in production function regressions, is effectively the firm's revenue, and not (physical) output. Thus, the estimated returns to scale are returns in *revenue*, not production. This robust result has important implications for the estimation and interpretation of returns to scale.

I show that if firms face perfectly competitive factor markets, a common assumption, then returns to scale (RTS) in the revenue function cannot exceed unity otherwise the profit share in revenue is negative. Least squares estimates of RTS frequently exceed unity (e.g., Griliches and Ringstad 1971, Tybout and Westbrook 1996, Bartelsman and Dhrymes 1998), a finding can be explained by the transmission bias identified by Marschak and Andrews (1944). Yet, empirical estimates of RTS often exceed unity even after correcting for the transmission bias (e.g., Pavcnik 2002, Levinsohn and Petrin 2003). In other words, these estimates suggest that firms systematically violate the profit maximization principle. Thus, finding a convex revenue function raises legitimate concerns about the validity of the applied economic model and statistical estimator.

At the other extreme, studies often find low returns to scale in the revenue function. If factor markets are competitive, these low returns to scale in the revenue function imply a much larger profit share in revenue than is observed in the data. For example, 0.8 returns in the revenue function entails a profit share of 20% in revenue (or 50% in value added if the share of materials is 0.6). In most data the profit share is 3% or less (Rotemberg and Woodford 1995, Basu and Fernald 1997). Griliches and Hausman (1986) attribute low returns to large, (possibly) serially correlated measurement errors, which are hard to handle in the instrumental variables (IV) framework as there are few good instruments to cope with measurement errors. It is not clear, however, why measurement errors are so pervasive in some industries and not others.

To address these problems, I consider a cost-minimizing firm that can face upward- or downward-sloping factor supply curves. Under weak assumptions, I link the profit share, the elasticity of the cost with respect to inputs, returns to scale in production and the markup. I suggest that the profit share can serve as a robust non-parametric diagnostic for checking if the estimates of production functions make economic sense. Using this model, I reconcile increasing returns to scale in the revenue function and a small profit share. Likewise, I can explain large decreasing returns to scale and small profit share.

Unfortunately, available estimators either do not estimate the elasticity of the cost or depend critically on the assumption that factor supply curves are perfectly elastic. Furthermore, I identify empirically plausible cases when popular estimators of production functions are likely to perform poorly and, thus, produce the puzzles. In particular, I argue that method of moments estimators such as Blundell and Bond (1999) can be poorly identified because of economic restrictions on the comovement of inputs and output in the context of estimating production functions.[1] I also argue that inversion estimators such as Olley and Pakes (1996), Pavcnik (2002), and Levinsohn and Petrin (2003) are inconsistent because they ignore variation in input mixes (e.g., variation in the materials/labor ratio). Similarly, if there is measurement error, the inversion estimators are invalid because there is no one-to-one inversion from observed choices of firms to their technology. Interestingly, least squares estimates of returns to scale in the revenue function are likely to have a relatively small bias.

I argue that simple structural estimators can address these problems and can provide an estimate of the elasticity of the cost. Specifically, I extend the full information maximum likelihood estimator of Marschak and Andrews (1944) and Schmidt (1988) to dynamic production function models with serially correlated measurement errors and factor prices correlated with productivity. This estimator, which I call the covariance estimator, deals simultaneously with production and cost sides and with unobserved technology and factor prices. The key idea of the estimator is to use the covariance structures for the firms' observed choices (inputs, outputs) to estimate parameters of the production function using the restrictions imposed by the economic model on the response of observed variables to unobservables such as productivity and factor prices. In some cases, the covariance estimator has an instrumental variable interpretation. I show that the covariance estimator outperforms popular alternatives in Monte Carlo experiments and yields economically more reasonable estimates than those from alternative estimators when confronted with the real data.

[1] This source of poor identification is different from issues arising in very persistent series.

More broadly, I argue that consistent estimation of production function parameters requires modeling not only unobserved technology but also unobserved factor prices and other structural shocks. Sweeping the latter variation under the "ceteris paribus" rug can greatly distort measures of productivity, resource reallocation, calibration of economic models, etc. It is equally important to model both revenue and cost side of optimizing firms. The compensation for extra effort in economic modeling for structural estimators is better understating of the firm's behavior.

The structure of the paper is as follows. In the next section, I present theoretical results and discuss the sources of identification in production functions and examine the variables used in the production function regressions. In Section 3, I present the covariance estimator and discuss identification and estimation issues. In Section 4, I derive the theoretical predictions about the performance of OLS, instrumental variables and inversion estimators. Monte Carlo experiments in Section 5 illustrate the performance of alternative estimators. In Section 6, I use a well-known Chilean firm-level data to compare RTS estimates from the covariance estimator and popular alternatives. I present conclusions in Section 7.

2. SETUP

In this section, I derive the relationship between the markup, returns to scale in production, the elasticity of the cost and the profit share. I demonstrate that the profit share can serve as a robust non-parametric diagnostic for economic tests of the estimates of production (revenue) functions.

2.1. ECONOMIC MODEL OF PRODUCER BEHAVIOR

Consider a firm that minimizes cost in expectation or non-stochastically. I assume that the cost of inputs is separable in inputs and factor prices, i.e., cross-partial derivatives of the cost with respect to factor prices and inputs are equal to zero. Hence, the cost can be written as $C(L,w)=\sum_{j=1}^{n} C_j(L_j,w_j)$ where L and w are vectors of inputs and factor prices, L_j is the j^{th} input and w_j is its price. The elasticity of the cost C_j with respect to input j is $\phi_j=\dfrac{\partial C_j(L_j,w_j)}{\partial L_j}\cdot\dfrac{L_j}{C_j(L_j,w_j)}$. The share of input j in total cost is $\omega_j = C_j(L_j,w_j)/C(L,w)$.

Returns to scale in production γ is defined as $\gamma=\sum_{j=1}^{n}(\partial Q/\partial L_j)L_j/Q(L)$ where $Q(L)$ is the production function. Analogously, returns to scale in the revenue function η is defined as

$\eta = \sum_{j=1}^{n} (\partial Y / \partial L_j) L_j / Y(L)$ where Y is total revenue. I define markup μ as the ratio of the output price to the marginal cost. Profit share is defined as $s_\pi = (Y - C)/Y$. Note that I make no assumptions about production function, structure of product and factor markets.

To simplify exposition, I also assume that firms can freely adjust factors of production to avoid unnecessary complications arising from dynamic optimization. This assumption implies that firms solve a *static* profit maximization problem in every period and inputs and output are chosen simultaneously. Alternatively, one can interpret this assumption as describing a large cross-section of firms or the long run when firms can adjust all inputs. In this general setup, the following result can be proven:

Proposition 1.

Suppose a firm minimizes cost, all inputs are variable, and its cost is separable in inputs. Then, $\gamma / \mu = (1 - s_\pi)\phi$, where μ is the markup, $\phi = \sum_{j=1}^{n} \phi_j \omega_j$ is the elasticity of the cost with respect to inputs, ϕ_j is the elasticity of the j^{th} factor cost, ω_j is the share of input j in total cost, γ is returns to scale in production, and s_π is the profit share in revenue. Furthermore, if the firm maximizes profit, then $\eta = \gamma / \mu$, where η is returns to scale in the revenue function.

Proof: see appendix C.

One can draw several conclusions from Proposition 1. First, consider the case where factor supplies are perfectly elastic (i.e., $\phi_j = 1$ for all j). Since the profit share s_π is close to zero (Rotemberg and Woodford 1995, Basu and Fernald 1997), by Proposition 1 the returns in the revenue function η, which is equal to γ / μ, should be approximately unity. Furthermore, industries with large returns to scale in production γ should have a large markup μ such that $\mu \approx \gamma$. Hence, finding constant returns to scale in revenue is very likely to indicate that there are increasing returns to scale in production since the markup is often greater than 1.05-1.1 (e.g., Bresnahan 1988) Proposition 1 shows that low returns to scale in the revenue function imply a large profit share. For instance, η=0.8 implies s_π=20%. Similarly, finding η>1 is not consistent with profit maximization since η>1 implies a negative profit share. More generally, if the profit share implied by an estimate of η is far from the profit share observed in the data, then one has a signal that either the statistical or economic model is incorrect. This point is first raised and proven in Basu and Fernald (1997). Because Proposition 1 makes weak assumptions about producer behavior, the profit share serves as a robust non-parametric economic diagnostic for statistical estimates of η.

Second, consider the case where factor supplies are not perfectly elastic (i.e., $\phi_j \neq 1$ for some j). In this case, there is no tight link between the profit share and returns to scale in the revenue function η. For example, increasing returns to scale in the revenue function η and a small positive profit share can be reconciled by a steep cost (i.e., large ϕ). For example, monopsony power or shift premium can result in an upward-sloping labor supply schedule. Likewise, decreasing returns to scale in revenue or production functions can be consistent with a small profit share if ϕ is less than unity, i.e., the marginal unit cost of inputs is (locally) declining. Table 1 summarizes the relationship between s_π, η and ϕ.

Note that in the case with $\phi \neq 1$ the cost-based Solow residual does not measure technology (or revenue generating ability) correctly because cost shares are not equal to the elasticities of output with respect to corresponding inputs. Specifically, the cost-based Solow residual then depends on factor ratios and, thus, it can be procyclical and serially correlated.

For the case with some inputs being fixed, the following Corollary to Proposition 1 can be proven:

Corollary 1

Suppose that the assumptions of Proposition 1 hold. Also suppose that first k inputs are variable and other n-k inputs are fixed. Then, $\gamma^* / \mu = (1 - s_\pi^*)\phi^* \omega^*$, *where* $\phi^* = \sum_{j=1}^{k} \phi_j \omega_j$ *is the elasticity of the cost with respect to variable inputs,* ω^* *is the cost share of variable inputs in total cost,* γ^* *is returns to scale in production with respect to variable inputs, and* s_π^* *is the profit share in revenue. Furthermore, if the firm maximizes profit, then estimated returns to scale with respect to variable inputs is* $\eta^* = \gamma^* / \mu$.

Proof: see appendix C.

Corollary 1 suggests that the argument about the profit share should be applied to variable inputs only. The corollary explains that the profit share can be temporarily large since γ^* can be significantly less than unity or temporarily small since the short term elasticity of the variable factor supplies can be low (i.e., ϕ^* large). Since there is no optimization with respect to fixed inputs, cross-sectional variation in the fixed inputs is sufficient to identify the returns to scale with respect to fixed inputs and, hence, returns to scale with respect to all inputs.

In summary, Proposition 1 justifies using the profit share as an economic check to verify that statistical estimates of production function parameters make economic sense. Put differently, since parameter ϕ can be interpreted as returns to scale in the cost, returns to scale in the revenue function η is always less than returns to scale in the cost but the difference is small. Furthermore, since the profit share is small, a consistent estimate of returns to scale in the revenue function can inform the researcher about the properties of the cost, specifically about its parameter ϕ. Likewise, one can infer η from ϕ.

2.2. FUNCTIONAL FORMS

To make further progress in the analysis of estimated returns to scale, I assume functional forms for production and demand function and cost.[2] Specifically, the inverse demand function is isoelastic $P_{it} = G_{it} \cdot Q_{it}^{-1/\sigma}$ where *i* and *t* index firms and time, P_{it} is the price of the good, Q_{it} is the quantity of the good, G_{it} is a demand shifter (e.g., quality of a good, macroeconomic conditions) and σ is the elasticity of demand. The markup is $\mu = \sigma/(\sigma-1)$. The production function is $Q_{it} = A_{it}^{\mu} Z_{it}^{\gamma}$ where A_{it} is Hicks-neutral firm-specific productivity (the power of A_{it} is a normalization to simplify notation), and Z_{it} is a composite input. For the case with multiple inputs, I assume a Cobb-Douglas production function. Here and henceforth, I assume that inputs are measured in physical units. The cost of consuming Z_{it} is $W_{it} Z_{it}^{\phi}$ where W_{it} is the base price of the input and ϕ is the elasticity of the cost with respect to the input Z_{it}. The case of $\phi = 1$ correspond to supply of Z_{it} being perfectly elastic. Hence, the profit is $\pi_{it} \equiv Y_{it} - W_{it} Z_{it}^{\phi}$ where $Y_{it} = P_{it} Q_{it}$ is the revenue function. The profit function is concave in the input if and only if $\gamma/\mu - \phi < 0$.

Taking logs of the first order conditions, suppressing uninteresting constants, and partialing out industry-wide shocks, one obtains the following expressions for optimal input, revenue, and price

[2] This model of producer behavior is similar to the model analyzed by Marschak and Andrews (1944) and Klette and Griliches (1996).

$$z_{it} = \frac{1}{\eta - \phi} w_{it} - \frac{1}{\eta - \phi}(a_{it} + g_{it}), \quad (1)$$

$$y_{it} = \frac{\eta}{\eta - \phi} w_{it} - \frac{\phi}{\eta - \phi}(a_{it} + g_{it}), \quad (2)$$

where small letters denote logs of the respective variables, and $\eta=\gamma/\mu$ is the returns to scale in the revenue function. Note that demand shocks G_{it} and technology shocks A_{it} are isomorphic and, thus, are not identified separately.[3] Henceforth, I treat G_{it} as it were a shock to technology and consider only A_{it}. It will be convenient in further analysis to write (1)-(2) in matrix form:

$$\boldsymbol{X}_{it} \equiv \begin{bmatrix} z_{it} \\ y_{it} \end{bmatrix} = \begin{bmatrix} \frac{1}{\eta-\phi} & \frac{-1}{\eta-\phi} \\ \frac{\eta}{\eta-\phi} & \frac{-\phi}{\eta-\phi} \end{bmatrix} \begin{bmatrix} w_{it} \\ a_{it} \end{bmatrix} \equiv \Lambda \boldsymbol{F}_{it} . \quad (3)$$

Equations (1) and (2) indicate that output and input demand are increasing in productivity a_{it} and decreasing in the factor price w_{it}. Note that one can consider (1)-(2) as a log-linear approximation of other function forms for production, cost and demand. Importantly, a_{it} and w_{it} are firm-specific price and productivity shocks because industry-wide shocks are partialled out. Since variation in technology a_{it} across firms is not controversial (e.g., Bartelsman and Doms 2000), in the next section I focus on w_{it} as a source of variation in (1)-(2).

2.3. ON SOURCES OF VARIATION

In the model (1)-(2), I use variation in the factor price w_{it} to address two stylized facts. First, inputs and output are not collinear in the data. Second, there is enormous variation in input mixes. For example, the interquantile (Q_3-Q_1) range of log(capital/labor) and log(materials/labor) for Chilean and U.S. manufacturing firms is typically above 100% even at four-digit SIC industries. Note that in any model that assumes Hicks-neutral technology such variation in input mixes can happen only if firms face different input prices or technology or firms cannot satisfy profit maximizing (cost minimizing) conditions (e.g., because of managerial errors). Hence, variation in w_{it} can explain both facts.

I am agnostic about the reasons why firms face different input prices. My non-exhaustive list includes unionization, regulation, location, composition of capital, and subjective beliefs of the management about factor prices. Search and information costs result in equilibrium price

[3] Under stronger assumptions it is possible to separate demand and technology shocks. For example, Katayama, Lu and Tybout (2003) assume Bertrand pricing and constant marginal cost to identify demand and technology shocks.

dispersion even if firms are identical ex ante (e.g., Stigler 1961, Salop and Stiglitz 1982, Burdett and Judd 1983, Stahl 1989).

There is substantial direct evidence on the dispersion of prices even for precisely defined products (Stigler 1961, Pratt, Wise and Zeckhauser 1979, Dahlby and West 1986, Abbott 1992, Sorensen 2000). Using firm-level U.S. Census data, Abbott (1992) reports that the mean coefficient of variation for output prices at 7-digit product codes is at least 55% (see also Roberts and Supina 1996). Even prices of homogenous inputs such as cement have significant dispersion at local markets (Abbott 1992, Adams 1997, Lach 2002, Yoskowitz 2002). For 70% of firms, other firms are the main customers (Fabiani et al 2004) and, thus, such price dispersion is an important source of variation in input mixes.

Likewise, there is voluminous evidence that similar workers are paid different wages (e.g., Mortensen 2003 and references cited therein). Abowd, Creecy and Kramarz (2002) find that approximately 40-50% of wage dispersion in France and the state of Washington is determined by firm effects. Price dispersion in capital/financial markets is less documented yet it exists (see Hortaçsu and Syverson (2004) for an example of dispersion of fees charged by mutual index funds). Multiplicity of interest rates also suggests that different firms face different prices of capital even within the same industry and location. Furthermore, firms may have different *shadow* prices of inputs (because of adjustment costs, for example) even when they face the same posted market prices for inputs.

There are alternative explanations for variation in input mixes. In the early studies of production functions (e.g., Marschak and Andrews 1944, Hoch 1961, Zellner, Kmenta and Dreze 1966), it is assumed that managerial errors determine the variation in input ratios.[4] In another interpretation (e.g., Stigler 1976, McElroy 1987), managerial errors reflect constraints known to the management but unobserved to the econometrician.[5]

[4] For example, Zellner, Kmenta and Dreze (1966, p. 785) assume that, "The random disturbance terms in the traditional model [of non-stochastic first order conditions] are introduced to allow for random, nonsystematic errors on the part of entrepreneurs in their attempts to adjust inputs to satisfy the necessary conditions for profit maximization."

[5] Another explanation of variability in input mixes is variation in parameters of the production function. A typical approach to estimate models with parameter heterogeneity (e.g., Mairesse and Griliches 1990, Biorn, Lindquist and Skjerpen 2002) is to use the random coefficients estimator (Swamy 1970) that assumes zero covariance between random coefficients and regressors. This assumption is, however, clearly violated in the context of production functions if management knows the parameters of its production function. Consider the model in (1)-(2) with no measurement errors, $\mu = 1$ and random firm-specific RTS parameter γ_i such that $\gamma_i \sim iid(\overline{\gamma}, \sigma_\gamma^2)$ and

Although the managerial errors theory may be right, it can hardly explain immense variation in input mixes. (Recall that the interquantile range of log input ratios is generally above 100%.) In addition, all measures of dispersion for input ratios increase with aggregation. It is hard to reconcile these facts with managerial errors theory because there is no reason to expect that managerial errors become more important with aggregation. In contrast, variation in prices for labor, capital and materials gives an estimate of volatility in input mixes in the right ball park.[6]

Differences in interpretation, however, do not generally imply differences in estimates of RTS. For example, suppose that factor prices are the same across firms and consider a Cobb-Douglass production function with labor L_{it} and capital K_{it} inputs and managerial errors $\zeta_{it}^K, \zeta_{it}^L$ in the first order conditions $\beta_K Y_{it}/K_{it} = R_t \exp(\zeta_{it}^K)$ and $\beta_L Y_{it}/L_{it} = W_t \exp(\zeta_{it}^L)$, where β_K and β_L are elasticities of the revenue function with respect to capital and labor, Y_{it} is the revenue, R_t is the cost of capital and W_t is wages. After taking logs and ignoring uninteresting constants, one has $y_{it} = k_{it} + \zeta_{it}^K$ and $y_{it} = l_{it} + \zeta_{it}^L$. If one assumes firm-specific factor prices, the corresponding first order conditions are $y_{it} = k_{it} + r_{it}$ and $y_{it} = l_{it} + w_{it}$. Thus, the models are observationally equivalent and give identical estimates of parameters in the revenue function. As a result, I will treat factor prices as generic shocks to input ratios.

2.4. WHAT "PRODUCTION FUNCTION" REGRESSIONS ESTIMATE?

Firm-level data sets (e.g., Longitudinal Business Database at the U.S. Census Bureau) rarely contain information about prices paid/charged by firms or quantities consumed/produced by

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$\mathrm{cov}(w_{it}, \gamma_i) = \mathrm{cov}(a_{it}, \gamma_i) = 0$. The estimated model is $y_{it} = \gamma_i z_{it} + u_{it}$. It is not hard to find $\mathrm{cov}(x_{it}, \gamma_i) \approx -[\gamma^2(\gamma-1)(\gamma-2) + \sigma_\gamma^2]/(\gamma-1)^2 < 0$. Because $\mathrm{cov}(x_{it}, \gamma_i) \neq 0$, the estimator is not consistent and results should be interpreted very carefully.

[6] For example, Abowd, Creecy and Kramarz (2002) report that the standard deviation of log real hourly wages is 53%. If one takes the coefficient of variation as a proxy for the standard deviation of log deviations from the mean, then the standard deviation of material prices is 55% (Abbott 1992) at the 7-digit level. At the 4-digit level, the standard deviation is likely to be several times larger. Hence, variation in the ratio of prices for labor and materials, which is equal to log(labor/materials), can be as large as 100%. Likewise, the standard deviation of log fees in mutual funds is about 50% (Hortaçsu and Syverson 2004), which, however, can be an upper bound. Hence, variation in log wage to capital price, which is equal to log labor to capital ratio, can also be as volatile as 100%. Of course, the observed variation can be endogenous, yet this calculation is suggestive. Note that this simple calculation ignores possible variation in shadow prices which can be considerably larger than the variation in posted prices because shadow prices can differ across firms due to adjustment costs, complementarity of inputs, composition of inputs (especially vintages of capital), etc.

firms. In vast majority of cases, the econometrician observes only inputs and revenue of the firm and, hence, a typical production function regression is

$$y_{it} - \overline{p}_t = bz_{it} + u_{it} , \tag{4}$$

where $\overline{p}_t$ is industry price index, b is estimated returns to scale, u_{it} is the error term, and the dependent variable is the firm revenue deflated by industry price index.[7] In the standard framework of monopolistic competition (Dixit and Stiglitz 1977), the demand function is $p_{it} = \overline{p}_t - \frac{1}{\sigma} q_{it} + g_{it} + const$ and, the hence,

$$\begin{aligned} y_{it} - \overline{p}_t &= (p_{it} + q_{it}) - \overline{p}_t = (1 - \tfrac{1}{\sigma}) q_{it} + g_{it} + const = \\ &= (1/\mu)(\gamma z_{it} + \mu a_{it}) + g_{it} + const = \eta z_{it} + (a_{it} + g_{it}) + const . \end{aligned} \tag{5}$$

Clearly, the coefficient b in (4) reflects returns in the revenue function η, not returns to scale in production γ. Furthermore, because firms face different productivity and/or wage realizations, the price of the good varies across firms and, as I discussed in the previous section, dispersion of prices is not trivial even in narrowly defined industries. Therefore, deflating the firm's revenue with an industry price index $\overline{p}_t$ does not generally yield the firm's output. In the limiting case where the share of the firm in industry output converges to zero and shocks to productivity and factor prices are not perfectly correlated across firms (recall that industry-wide shocks are partialled out), the cross-sectional variation of $(y_{it} - \overline{p}_t)$ converges to the cross-sectional variation in y_{it}, i.e., the dependent variable in typical firm-level production function regressions is effectively the firm's revenue y_{it}, not the firm's output q_{it}.[8]

One has to be careful with the interpretation of the residual in (4) as well. Note that the error term in (5) combines demand shocks G_{it} and technology shocks A_{it} and, hence, one should not attribute large residuals to high technology because a large residual can stem from a large demand shock. Likewise, large variation of u_{it} in (4) should not be interpreted as large variation in technology.

[7] Foster, Haltiwanger and Syverson (2005) is an important exception. They consider firms producing homogenous goods so that information on revenue and physical output is available.

[8] See Klette and Griliches (1996) for further discussion. Also note that time dummies are often included in (4) so that deflation by $\overline{p}_t$ is irrelevant.

2.5. *RECAPITULATION*

This section makes several points. First, because of data limitations, typical production function regressions based on firm level data use revenue as the dependent variable and, hence, estimate returns to scale in the revenue function and do not yield the Solow residual measuring technical efficiency of firms. Second, profit share in the revenue s_π should be used as a robust nonparametric diagnostic for the estimates of RTS in the revenue function. Third, increasing or decreasing RTS in the revenue function and a small profit share s_π can be reconciled by ϕ, the elasticity of the cost with respect to inputs. Hence, the parameter ϕ is of central importance. Fourth, there is sizable variation in factor prices across firms.

Unfortunately, available estimators either do not yield an estimate of ϕ or hinge critically on the assumption that $\phi = 1$ (see Section 4). To address this problem, I develop a full-information estimator that deals with production and cost sides simultaneously.

3. COVARIANCE ESTIMATOR

To consistently estimate parameters of production/revenue and cost functions, I suggest an estimator based on explicit specification and modeling of unobserved shocks (i.e., productivity, demand, wages, etc.) where factor price shocks are treated symmetrically with productivity shocks, instead of just focusing on productivity shocks. The idea of the estimator is to identify and estimate parameters of the model by matching covariance matrix implied by the model to the empirical covariance matrix of observed choices of firms. In contrast to single equation estimators (e.g., OLS), this structural estimator models outputs and inputs simultaneously (system approach) by deriving optimal output and factor demands from a profit maximization or cost minimization problem. This estimator falls under the rubric of structural equation modeling (see Bollen 1989 for a general discussion).[9] In this section I explain the intuition behind the estimator and discuss identification and estimation.

3.1. *INTUITION*

To illustrate the workings and intuition of the estimator, consider model (1)-(2) and assume—for reasons discussed later—that $\phi = 1$ and a_{it} and w_{it} have variances σ_a^2 and σ_w^2 with $\rho(a_{it}, w_{it}) = 0$.

[9] This approach is also called MIMIC, LISREL and other names.

These assumptions are restrictive and later I will show that the estimator works under less stringent conditions.

Because a_{it} and w_{it} are not observed, one cannot run a regression of z_{it} and/or y_{it} on these shocks to estimate the returns to scale in the revenue function η. Note, however, that $\mathrm{var}(z_{it}) = (\eta-1)^{-2}(\sigma_w^2+\sigma_a^2)$, $\mathrm{var}(y_{it}) = (\eta-1)^{-2}(\eta^2\sigma_w^2+\sigma_a^2)$, and $\mathrm{cov}(y_{it},z_{it}) = (\eta-1)^{-2}(\eta\sigma_w^2+\sigma_a^2)$ with unknowns $\eta,\sigma_a^2,\sigma_w^2$. Since the second moments are observed, one can solve the system of equation for η:

$$\eta = \frac{\mathrm{var}(y_{it}) - \mathrm{cov}(y_{it},z_{it})}{\mathrm{cov}(y_{it},z_{it}) - \mathrm{var}(z_{it})}. \tag{6}$$

Thus, one can estimate η from the second moments of the data. This was the insight of the seminal paper by Marschak and Andrews (1944). I will call (6) and expressions analogous to (6) the covariance (COV) estimator. Why is the estimator working?

Equations (1)-(2) describe the optimal profit-maximizing behavior of firms and optimization imposes restrictions on how firms respond to shocks. Specifically, the assumption of Hicks-neutral technology and perfectly elastic factor supply curve result in the restriction that revenue and input demand respond equally strongly to an innovation in technology. In other words, the coefficient on the structural shock a_{it} is the same in equations (1) and (2). Furthermore, the assumption of the perfectly elastic factor supply curve implies the restriction that the response of revenue to a shock in the factor price w_{it} is η time stronger than the response of the factor demand z_{it} to the factor price shock. Put differently, the coefficient on w_{it} in equation (2) is equal to the coefficient on w_{it} in (1) multiplied by η. The economic restrictions of Hicks-neutral technology and perfect elasticity of the factor supply are complemented with the technical restriction $\rho(a_{it},w_{it})=0$. This latter condition ensures that one can separate technology shocks and factor price shocks. If technology and factor prices are correlated, this simple model is not identified.

This estimator can have an instrumental variables interpretation. Equation (6) can be equivalently written as

$$\eta = \frac{\mathrm{cov}(y_{it}, y_{it}-z_{it})}{\mathrm{cov}(z_{it}, y_{it}-z_{it})} \tag{7}$$

and, hence, y_{it}-z_{it} is an instrumental variable for z_{it}. Because of the Hicks-neutral technology and perfectly elastic factor supply, profit maximization imposes that revenue y_{it} and input z_{it} respond equally strongly to productivity shocks a_{it} and, hence, $y_{it} - z_{it} \propto w_{it}$. Given the assumption $\rho(a_{it}, w_{it}) = 0$, y_{it}-z_{it} is correlated with z_{it} and uncorrelated with a_{it}. In this simple case, covariance and instrumental variable estimators are equivalent. However, as I will discuss below, explicit instrumental variables like y_{it}-z_{it} are not always available and typically the instrument depends on an unknown parameter.

3.2. MODEL FRAMEWORK

The basic model (3) can be generalized along several dimensions. First, I specify the dynamics of unobserved technology and factor prices collected in the vector $\boldsymbol{F}_{it}$. Second, measurement errors are salient in micro-level data sets. To address this important fact, I augment (3) with measurement errors. Third, I allow observed choices of firms to respond to observed exogenous variables. In summary, the general model is

$$\boldsymbol{X}_{it} = \Lambda \boldsymbol{F}_{it} + \bar{\boldsymbol{X}}_i + \boldsymbol{\varepsilon}_{it}, \tag{8}$$

$$\boldsymbol{F}_{i,t+1} = \Pi \boldsymbol{F}_{it} + \boldsymbol{\upsilon}_{it}, \tag{9}$$

where $\boldsymbol{X}_{it}$ is the vector of n observed variables (inputs and revenue), $\boldsymbol{F}_{it}$ is the vector of m unobserved variables (factor prices, productivity), the matrix Λ summarizes the responses of observed variable to $\boldsymbol{F}_{it}$, $\bar{\boldsymbol{X}}_i$ is a vector of unobserved permanent firm-specific effects for $\boldsymbol{X}_{it}$, $\boldsymbol{\varepsilon}_{it}$ is a vector of i.i.d. zero-mean measurement or expectations errors, $\boldsymbol{\upsilon}_{it}$ is a vector of i.i.d. structural zero-mean innovations to $\boldsymbol{F}_{it}$, and the matrix Π captures the dynamics of $\boldsymbol{F}_{it}$.[10] The matrix Λ for the n-input case is given in (A.1), Appendix A. I collect parameters of the model in the vector θ and assume here and henceforth that the mapping from θ to $\Pi, \Lambda, \Omega \equiv E(\boldsymbol{\upsilon}_{it}\boldsymbol{\upsilon}'_{it}), \Psi \equiv E(\boldsymbol{\varepsilon}_{it}\boldsymbol{\varepsilon}'_{it})$ is one-to-one in the admissible domain of θ.

This state space representation of the problem nests many important cases such as dynamic factor models ($m<n$), log-linearized rational expectations models in state-space form and serially correlated measurement errors.[11] I am agnostic about time series properties of $\boldsymbol{F}_{it}$ and

[10] Since dependence of $\boldsymbol{X}_{it}$ on observed exogenous variables (e.g., time dummies) can be easily eliminated by projection methods, I abstract from such dependence without loss of generality.

[11] This case is important in practice because econometricians rarely have reliable estimates of capital stock, effort, etc. For example, there are two popular estimates of capital: 1) real capital stock computed by inventory methods; 2)

contemporaneous correlation of innovations in $\boldsymbol{\upsilon}_{it}$ as economic theory may have few restrictions on how variables in $\boldsymbol{F}_{it}$ evolve over time or how $\boldsymbol{\upsilon}_{it}$ is correlated. Note that variables in ***Fit*** can correlated because either Π or Ω is not diagonal. Likewise, I do not impose any structure on $\bar{\boldsymbol{X}}_i$.

The model (8)-(9) has much in common with dynamic factor models. However, in contrast to dynamic factor models, the factor loadings embodied in the matrix Λ can be identified under certain conditions and, thus, factors can have structural interpretation. In the next section, I present the conditions under which θ is identified.

3.3. IDENTIFICATION

The key question for the COV estimator is the identification of parameters because many models can be consistent with observed covariances. Local identification of these parameters in the static model (8) and dynamic model (8)-(9) is discussed elsewhere (e.g., Hoch 1958, Maravall and Aigner 1977, Maravall 1979, Bollen 1989, Bekker, Merkens and Wansbeek 1994). In effect, local identification requires showing that the Jacobian of the objective function has full rank. Obviously, the necessary condition for identification is that the number of parameters in θ is not greater than the number of unique moments in considered covariance and autocovariance matrices.

Global identification is more subtle. In factor analysis terminology, global identification reduces to proving that there is no rotation matrix T producing $\{\tilde{\Lambda},\tilde{\Pi},\tilde{\Omega},\tilde{\Psi}\} = \{\Lambda T, T^{-1}\Pi T, \Psi, T^{-1}\Omega T'^{-1}\}$ observationally equivalent to $\{\Lambda,\Pi,\Omega,\Psi\}$. Fortunately, profit maximization imposes many restrictions on the matrix Λ and, hence, global identification can be ensured under relatively mild assumptions.

Global identification is more subtle. In factor analysis terminology, global identification reduces to proving that there is no rotation matrix T producing $\{\tilde{\Lambda},\tilde{\Pi},\tilde{\Omega},\tilde{\Psi}\} =$

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book value of fixed assets. In either case, measurement error is likely to be serially correlated. Suppose that the econometrician uses a noisy measure of investment such that e_t, the error in true investment I_t^*, is classical (the measurement error can arise from using investment price index to deflate firm-level investment expenditures). The true capital stock evolves according to $K_t^* = (1-\delta)K_{t-1}^* + I_t^*$. Then the estimated capital stock is $K_t = (1-\delta)K_{t-1} + I_t = K_t^* + \sum_{s=0}^{\infty}(1-\delta)^s e_{t-s} = K_t^* + \varepsilon_t^k$ with $\varepsilon_t^k = (1-\delta)\varepsilon_{t-1}^k + e_t$, that is, measurement error $\varepsilon_t^k \sim AR(1)$. Importantly, serially correlated measurement errors invalidate instrumental variables based on leads/lags of inputs/outputs or input mixes. Similarly, true labor input may be measured with serially correlated error because of labor hoarding.

$\{\Lambda T, T^{-1}\Pi T, \Psi, T^{-1}\Omega T'^{-1}\}$ observationally equivalent to $\{\Lambda, \Pi, \Omega, \Psi\}$ (see Theorem 5 in Tse and Anton 1972). Profit maximization imposes many restrictions on the matrix Λ and, thus, on admissible rotation matrices T yet these restrictions do not eliminate rotational equivalence in (8) -(9). Further restrictions on Ω and Π can guarantee identification. The following proposition proves global identification for two important special cases.

Proposition 2

Assume that

i) *the matrix* Π *is invertible,*

ii) *the eigenvalues of* Π *are in the unit circle*,

iii) *the system in* (8)-(9) *is observable and controllable,*

iv) $E(\boldsymbol{\varepsilon}_{it}) = E(\boldsymbol{\upsilon}_{it}) = 0$ and $E(\boldsymbol{\upsilon}_{it}\boldsymbol{\upsilon}'_{jt}) = E(\boldsymbol{\upsilon}_{it}\boldsymbol{\upsilon}'_{is}) = E(\boldsymbol{\varepsilon}_{it}\boldsymbol{\upsilon}'_{jp}) = E(\boldsymbol{\varepsilon}_{it}\boldsymbol{\varepsilon}'_{is}) = E(\boldsymbol{\varepsilon}_{it}\boldsymbol{\varepsilon}'_{jt}) = 0$ *for any t,i,p,j and* $s \neq t$ *,*

v) *firms maximize profits so that the matrix of loadings* Λ *is as in* (A.1)*,*

vi) *at least one of the factors is supplied in a competitive market.*

Then the model (8)-(9) *is uniquely globally identified if*

a) *innovations in* $\boldsymbol{\upsilon}_{it}$ *are contemporaneously uncorrelated* (*that is, the covariance matrix* Ω *is diagonal*)*, or*

b) *the matrix* Π *is diagonal* (*that is, there are no dynamic cross-variable responses*)

Proof: see Appendix C.

The assumption that one of the factors is supplied in a perfectly competitive market fixes the elasticity of the factor supply curve for other inputs which, in turn, fixes the parameters of the revenue function. Note that factor price and productivity can be correlated in both *a*) ob *b*). Identification is achieved by imposing restrictions on the correlation of innovations in factor prices and technology (Ω is diagonal) or by imposing restrictions on the propagation of the shocks to technology and factor prices (Π is diagonal). It is also possible to identify θ if combinations of restrictions on Π or Ω are available.[12]

Local identification of models with serially correlated measurement error is discussed in, e.g., Maravall (1979) and Maravall and Aigner (1977). In the next proposition, I present conditions under which the model is globally identified.

Proposition 3

Suppose that i) serially correlated measurement errors in observed inputs and outputs are not correlated across inputs and outputs at all leads and lags; ii) measurement errors are not correlated with factor prices and productivity and the number of serially

[12] Glover and Willems (1974) show that one needs to modify the conditions slightly if observed and latent variables can respond contemporaneously for the same set of shocks.

correlated measurement errors k cannot exceed the number of observed variables n; iii) serially correlated measurement errors are AR(1) and covariance stationary. Then Λ, Π, and Ω identified almost everywhere if Λ, Π, and Ω are identified in the absence of measurement errors.
Proof: see Appendix C.

Note that in Propositions 2 and 3 I use only time series variation in factor prices and technology to identify parameters of the model. In other words, I do not use variation in $\bar{\boldsymbol{X}}_i$. However, it is possible to use restrictions on the distribution of $\bar{\boldsymbol{X}}_i$ to achieve identification in otherwise underidentified model. For example, one may be willing to impose $\bar{\boldsymbol{X}}_i = \Lambda \bar{\boldsymbol{F}}_i$ with $Var(\bar{\boldsymbol{F}}_i)$ being diagonal. Such restrictions can be particularly important if between variation is large relative to the within variation.[13]

3.4. ESTIMATION AND INFERENCE

Without loss of generality I assume that the panel of the firms is balanced with t=0,…,T observations for each cross-section. The number of i.i.d. cross-sections is N. I collect the parameters of interest in the vector θ, which is locally identified. I assume that $\boldsymbol{X}_{it}$ is stationary. The estimation strategy is to find θ minimizing the distance between the appropriate sample covariance matrix and the covariance structure implied by θ. There are many possible ways to construct a metric of discrepancy between the sample and implied covariance matrices. I focus on maximum-likelihood methods since they tend to have somewhat better performance in finite samples because MLE does not use a weighting matrix that depends on unknown parameters (e.g., Clark 1996). I assume that the parameters are local identified.

It is convenient for further derivations to stack observed choices for each firm in vector $\boldsymbol{X}_i = [\boldsymbol{X}'_{i0} \quad \boldsymbol{X}'_{i1} \quad \ldots \quad \boldsymbol{X}'_{iT}]'$ where $\boldsymbol{X}_{it} = \bar{\boldsymbol{X}}_i + \Lambda \boldsymbol{F}_{is} + \boldsymbol{\varepsilon}_{is}$ and $\boldsymbol{F}_{it} = \Pi \boldsymbol{F}_{i,t-1} + \boldsymbol{\upsilon}_{it}$. Suppose that $\boldsymbol{\upsilon}_i = [\upsilon'_{0i} \quad \upsilon'_{1i} \quad \ldots \quad \upsilon'_{Ti}]' \sim N(0, \Omega \otimes I_T)$ and $\boldsymbol{\varepsilon}_i = [\varepsilon'_{0i} \quad \varepsilon'_{1i} \quad \ldots \quad \varepsilon'_{1T}]' \sim N(0, \Psi \otimes I_T)$ (i.e., measurement error $\boldsymbol{\varepsilon}_{it}$ and structural shocks $\boldsymbol{\upsilon}_{it}$ are normally distributed and serially uncorrelated)

[13] In applications, it may happen that η, returns to scale in the revenue function, is identified while other parameters in θ are not. In such cases, one can impose fairly arbitrary restrictions on unidentified parameters to have a well-defined estimation problem without affecting the identification of η (see Bollen 1989 for a discussion). If η is identified locally but not globally, it may be possible to rule out implausible cases, e.g., η<0. If η is not locally identified, one can follow Marschak and Andrews (1944) and put economic bounds on possible values of η. This amounts to constructing the set of values that parameters can take for all admissible rotations.

and $E(\boldsymbol{\upsilon}_i\boldsymbol{\varepsilon}_i')=0$ (i.e., structural shocks and measurement errors are not correlated at all leads and lags). Provided $\bar{\boldsymbol{X}}_i=0$, one can find that $\boldsymbol{X}_i \sim N(0,\Phi_T)$ where

$$\Phi_T \equiv E(\boldsymbol{X}_i\boldsymbol{X}_i') = \begin{bmatrix} \Sigma_0 & & & \\ \Sigma_1 & \ddots & & \\ \vdots & \ddots & \ddots & \\ \Sigma_T & \cdots & \Sigma_1 & \Sigma_0 \end{bmatrix} = \begin{bmatrix} \Lambda\Gamma_0\Lambda'+\Psi & & & \\ \Lambda\Gamma_0\Pi\Lambda' & \ddots & & \\ \vdots & \ddots & \ddots & \\ \Lambda\Gamma_0\Pi^T\Lambda' & \cdots & \Lambda\Gamma_0\Pi\Lambda' & \Lambda\Gamma_0\Lambda'+\Psi \end{bmatrix}, \quad (10)$$

with $\Sigma_s = E(\boldsymbol{X}_{it}\boldsymbol{X}_{i,t-s}')$, $\Gamma_s = E(\boldsymbol{F}_{it}\boldsymbol{F}_{i,t-s}')$, $\Gamma_0 : \Gamma_0 = \Pi\Gamma_0\Pi' + \Omega$. To simplify the notation, I use Φ instead of $\Phi(\theta)$, which explicitly indicates that Φ is a function of parameters collected in the vector θ. Hence, the likelihood function is given by

$$\sum_{i=1}^{N} l(\boldsymbol{X}_i,\theta) = \ln|\Phi_T| + trace\{\hat{\Phi}_T\Phi_T^{-1}\} - \ln|\hat{\Phi}_T| - Tn \quad (11)$$

where $\hat{\Phi}_T = \frac{1}{N}\sum_{i=1}^{N}\boldsymbol{X}_i\boldsymbol{X}_i'$, n is the number of observed choices of firms and maximum likelihood estimate of θ maximizes (11). Since rational expectations models can be represented in the state-space form like (8)-(9), it is an easy step to extend (11) to estimation of rational expectations models (see appendix A).[14]

For the case where steady state levels of inputs and output are treated as random, suppose that $\bar{\boldsymbol{X}}_i \sim N(0,\Xi)$ and $E(\bar{\boldsymbol{X}}_i\boldsymbol{u}_i')=0$ and observe that $\Phi_T = E(\boldsymbol{X}_i\boldsymbol{X}_i') = (\Xi \otimes J_T J_T') + \Phi_T$, where J_T is the $(T+1)\times 1$ vector of ones. It is straightforward to find that the associated likelihood satisfies $\sum_{i=1}^{N} l(\boldsymbol{X}_i,\theta) \propto -\ln|\Phi_T| - trace\{\hat{\Phi}_T\Phi_T^{-1}\}$. If $\bar{\boldsymbol{X}}_i$ is treated as a fixed parameter, one can transform the data to eliminate the incidental parameters $\bar{\boldsymbol{X}}_i$, e.g., apply first differencing as in Hsiao, Pesaran, and Tahmiscioglu (2002). The log-likelihood for first-differenced $\boldsymbol{X}_i$ satisfies: $\sum_{i=1}^{N} l(D\boldsymbol{X}_i,\theta) \propto -\ln|D\Phi_T D'| - trace\{(D\hat{\Phi}_T D')(D\Phi_T D')^{-1}\}$ where D is the $nT \times n(T+1)$ first-difference matrix. Alternatively, one can use conditional likelihood approach, which under certain conditions is equivalent to applying a transformation (e.g., Arellano 2003).

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[14] A popular alternative is generalized method of moments (GMM) with the objective function $J = N[\hat{\Phi}_T^* - \Phi_T^*(\theta)]'W^{-1}[\hat{\Phi}_T^* - \Phi_T^*(\theta)]$ where $\Phi_T'^* \equiv [vech(\Sigma_0)' \quad vec(\Sigma_1)' \quad \dots \quad vec(\Sigma_q)']$, $\hat{\Phi}_T^*$ is a sample estimate of Φ_T^*, W is a weighting matrix of conformable size. GMM and ML are asymptotically equivalent (Anderson and Amemiya 1988). If factor prices and productivity are uncorrelated, GMM and MLE are equivalent to IV estimator with (if necessary, leads or lags of) input ratios as instruments (Schmidt 1988).

Since $\boldsymbol{X}_i$ is not necessarily normally distributed, one may want to use the standard quasi-maximum likelihood tools to constructs standard errors for the estimates, i.e., $Var(\hat{\theta}) = N^{-1}H^{-1}GH^{-1}$ where $H = N^{-1}\sum_{i=1}^{N}\nabla_{\theta}^{2}l$ and $G = N^{-1}\sum_{i=1}^{N}\nabla_{\theta}l\cdot\nabla_{\theta}l'$. In the course of specification searches, one can use overidentifying restrictions tests since dynamic models such as (8)-(9) are typically overidentified. Importantly, it has been shown that specification tests based on likelihood ratios are sensitive to non-normality (see Bollen 1989 for discussion). Hence, one should evaluate the distribution of the test statistic using bootstrap procedures or rely on the statistic that is robust to non-normality.[15]

3.5. *DISCUSSION*

The structural approach embodied in the suggested estimator is built on earlier works on Marschak and Andrews (1944) and Schmidt (1988). I extend their static full-information maximum likelihood (FIML) estimators to dynamic settings and improve upon their FIML in several respect. First, I allow factor prices to be correlated with technology. This correlation can arise because of profit sharing, complementarity of worker skills and technology, monopsony power, overtime premia, etc. In contrast, the static models considered in previous studies are not identified if a_{it} and factor prices are correlated. Second, my extension permits classical and serially correlated measurement errors while static FIML is not identified if there is any measurement error. Third, I show that static and dynamic models can be identified and estimated when factor markets are imperfectly competitive, i.e., factor supply curves are not perfectly elastic. Specifically, I show that having an input with a perfectly elastic factor supply is sufficient for identification. Furthermore, I show in Appendix A that the covariance estimator can be extended to cases where the profit-maximizing firm faces adjustment costs.

There is a cost of using the covariance estimator. Like any other FIML estimator, the COV estimator is more sensitive to misspecification than single-equation methods (e.g., OLS). Since the COV estimator works with higher moments, it may be more sensitive to outliers.

[15] Monte-Carlo experiments (not reported here) suggest that finite sample performance of the COV estimator can be improved if a relatively small number of moments (sufficient for identification) are used in estimation. This enhancement is possible because low-order autocovariances can be estimated more precisely than in the presented formulation. For example, the first-order autocovaraince can be estimated using NT observations while in the presented formulation only N observations are used for the estimation. This issue is similar to choosing optimal number of moments in GMM application and it is left for future research.

4. ALTERNATIVE ESTIMATORS OF THE PRODUCTION FUNCTION

In this section I analyze alternative estimators of production function. I start with OLS to highlight the problems of estimating production functions and then proceed with the analysis of popular solutions to these problems. To contrast estimators, I use the dynamic model (8)-(9) with observed input *z* and output (revenue) *y*, measurement errors $\varepsilon_{it}^{z},\varepsilon_{it}^{y}$, and unobserved factor price *w* and technology *a*:

$$z_{it} = \tfrac{1}{\eta-\phi} w_{it} - \tfrac{1}{\eta-\phi} a_{it} + \varepsilon_{it}^{z} \tag{12}$$

$$y_{it} = \tfrac{\eta}{\eta-\phi} w_{it} - \tfrac{\phi}{\eta-\phi} a_{it} + \varepsilon_{it}^{y} \tag{13}$$

$$w_{i,t+1} = \rho_w w_{it} + \upsilon_{it}^{w} \tag{14}$$

$$a_{i,t+1} = \rho_a a_{it} + \upsilon_{it}^{a} \tag{15}$$

To simplify the presentation, I abstract from firm-specific effects. The estimated production function is

$$y_{it} = \eta z_{it} + a_{it} + \varepsilon_{it}^{y} = \eta z_{it} + error \tag{16}$$

This model makes exposition clear, yet my conclusions apply to more realistic cases as well.

4.1. OLS

Consider the producer as in model (12)-(15) and assume that variables are measured without error and a_{it} and w_{it} are uncorrelated i.i.d. zero-mean shocks with variances σ_a^2 and σ_w^2.[16] Using structural equations in (12)-(15), I find the probability limit of $\hat{\eta}_{OLS}$ in (16):

$$p\lim \hat{\eta}_{OLS} = \frac{\operatorname{cov}(y_{it},z_{it})}{\operatorname{var}(z_{it})} = \frac{\phi\sigma_a^2 + \eta\sigma_w^2}{\sigma_a^2 + \sigma_w^2} = \eta + (\phi-\eta)\frac{\sigma_a^2/\sigma_w^2}{1+\sigma_a^2/\sigma_w^2} > \eta$$

The upward bias in the OLS estimates is "the transmission bias" identified by Marschak and Andrews (1944). The asymptotic bias is decreasing in the variance of factor prices and, if the only source of variation is productivity, the OLS estimate is ϕ, the elasticity of the cost, irrespective of the true η, the elasticity of the revenue function.

[16] If firm-specific productivity is time invariant, then one can use panel data techniques to control productivity with firm-specific fixed effects (FE). This happy situation is not universally applicable and FE is not consistent if productivity is time varying. Furthermore, as Griliches and Mairesse (1995) observe, FE aggravates other problems (e.g., attenuation bias of measurement errors) precisely because of assuming time invariant differences in productivity across firms.

How big is the bias? If wage and productivity shocks are uncorrelated, then $bias = \frac{(\phi-\eta)\sigma_a^2}{\sigma_a^2+\sigma_w^2} < (\phi-\eta) = \phi s_\pi$ because $\phi-\eta = \phi s_\pi$ by Proposition 1. Since the profit share is 3% or less (e.g., Basu and Fernald, 1997) and ϕ is likely to be no greater than 1.5, the bias is positive but likely to be smaller than 0.045. Intuitively, the OLS estimate is between η and ϕ. Because these two quantities are close to each other, there is only a narrow range in which the OLS estimate can fall.[17] The same conclusion is likely to hold for cases with multiple inputs.[18] A relatively small bias in returns to scale, however, does not imply a small bias in the OLS estimate of the coefficient for a given input. Put differently, an upward bias in one of the coefficients is offset with a downward bias in other coefficients. Even if wage and productivity shocks are correlated, the asymptotic bias is likely to be small. This result, however, can be distorted by measurement errors.

4.2. IV/GMM ESTIMATORS

The transmission bias can be eliminated if the researcher has an instrumental variable (IV) explaining variation in z_{it} unrelated to productivity shocks a_{it}. In the simple setup of uncorrelated w_{it} and a_{it}, the best instrument is w_{it}, the price of z_{it}. The problem is that factor prices w_{it} are almost never collected and therefore such an IV is infeasible in the vast majority of cases. To rectify this problem, Schmidt (1988) suggests using input/output ratios as instruments, e.g., y_{it}-z_{it} in (7). If the production function is Cobb-Douglas, then Schmidt's IV (SIV) is identical to the IV estimator with factor prices as instruments. However, this SIV is not consistent if factor prices and productivity are correlated, the factor supply is not perfectly elastic (i.e., $\phi \neq 1$), or if either

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[17] If wage and productivity shocks are correlated, the asymptotic bias is $(\phi-\eta)(\sigma_a^2-\rho\sigma_a\sigma_w)/(\sigma_a^2+\sigma_w^2-2\rho\sigma_a\sigma_w)$ where $\rho=\rho(a_{it},w_{it})$. The OLS estimate of η can exceed ϕ if and only if $-\rho > \sigma_w/\sigma_a$ or fall below η if and only if $\rho > \sigma_a/\sigma_w$. The first case requires a negative correlation between productivity and factor price, which is somewhat implausible. The second case is more plausible but it still requires that productivity be less volatile than base wage. Bartelsman and Doms (2000) report that the ratio of the ninth decile of the distribution of a_{it} across firms to the first decile is typically about two to three. Juhn, Murphy and Pierce (1993) report that the log wage differential between the 90th and 10th percentiles in the private sector is about 1.5 and the differential is about 1.1 after controlling for observed labor force characteristics. Hence, σ_a/σ_w is likely to be large. If the correlation between a_{it} and w_{it} is in the range $(-\sigma_w/\sigma_a, \sigma_a/\sigma_w)$, the bound presented above is still appropriate, i.e., the bias is likely to be less than 0.045.

[18] The expression for the bias becomes complicated with multiple inputs and the upper bound for the bias depends on the elasticities of cost for specific inputs and relative variability of factor prices.

the output (revenue) or inputs are measured with a serially correlated error. Unfortunately, all of these cases are empirically important.

Alternatively, Blundell and Bond (1998, 1999, henceforth BB) suggest using 1) transformations of the variables to eliminate a_{it} from (16) and 2) lags of inputs and outputs as instruments. Specifically, BB suggest two types of moment conditions: levels and differences. Define $\vartheta_{it} \equiv y_{it} - \rho y_{i,t-1} - \eta z_{it} + \rho\eta z_{i,t-1}$, the residual from the quasi-differenced production function (16). The differences moment condition is $E(\Delta\vartheta_{it} g_{it}) = 0$ where g_{it} is any combination of $y_{i,t-j}, z_{i,t-j}, j \geq 3$. The levels moment condition is $E(\vartheta_{it} g_{it}) = 0$ where g_{it} is any combination of $\Delta y_{i,t-j}, \Delta z_{i,t-j}, j \geq 2$. Two options for estimation are available. First, estimate the unrestricted model (i.e., let $\vartheta_{it} \equiv y_{it} - b_1 y_{i,t-1} - b_2 z_{it} - b_3 z_{i,t-1}$ with b_1, b_2, b_3 being free parameters) and take the coefficient on z_{it} as $\hat{\eta}$. Second, estimate the restricted model.

The following result can be proven for the restricted specification:

Proposition 4

Consider profit-maximizing firms as in (8)-(9) *and estimate the production function using the restricted specification of the BB estimator. Then the model is not globally identified. In particular, the model has multiple locally-identified solutions, provided that the matrix* Π *has distinct eigenvalues. The number of solutions is no greater than n+1 where n is the number of inputs. If the matrix* Π *has repeated eigenvalues, then the model is not identified.*

Proof: see appendix C.

To get the intuition behind this result, consider, without loss of generality, the "levels" moments $E\{(y_{it} - \rho y_{i,t-1} - \eta z_{it} + \rho\eta z_{i,t-1}) z_t\} = 0$ where z_{it} is a subset of $\Delta y_{i,t-2}, \Delta y_{i,t-3}, \ldots, \Delta z_{i,t-2}, \Delta z_{i,t-3}, \ldots$. Use (12)-(13) to eliminate y_{it} and z_{it} from the moment condition and observe that two sets of parameter values satisfy the moment condition:

Solution #1: $\hat{\rho} = \rho_a, \hat{\eta} = \eta$ which yields $(y_{it} - \hat{\rho} y_{i,t-1} - \hat{\eta} z_{it} + \hat{\rho}\hat{\eta} z_{i,t-1}) = \upsilon_{it}^{a}$,

Solution #2: $\hat{\rho} = \rho_w, \hat{\eta} = 1$ which yields $(y_{it} - \hat{\rho} y_{i,t-1} - \hat{\eta} z_{it} + \hat{\rho}\hat{\eta} z_{i,t-1}) = \upsilon_{it}^{w}$.

In this simple case technology is Hicks-neutral and factor supply is perfectly elastic. Under these assumptions, profit maximization imposes that $y_{it} - z_{it} \propto w_{it}$ and $y_{it} - \eta z_{it} \propto a_{it}$. After appropriate quasi-differencing, each of these expressions is proportional to a serially uncorrelated shock. Thus, the objective function of the estimator in this simple case has two local minima.

In principle, the standard prescription is to choose a solution that gives the global minimum of some objective function (e.g., residual sum of squares), yet this heuristic may choose the incorrect solution #2. It may be hard to rule out some of the solutions on *economic* grounds. For example, in the presented one-input/one-output case, both solutions can be appealing. Furthermore, since the empirically observed profit share is small, η is likely to be close to unity (given perfect competition in factor markets) and, hence, estimator may be poorly identified even locally.

The consequences of having multiple solutions become particularly acute in the unrestricted specification since it is possible to take linear combinations of solutions such as above so that the model is not identified locally. The following proposition shows this formally.

Proposition 5

Consider profit-maximizing firms as in (8)-(9) or in a modification of (8)-(9) that allows for a cotemporaneous response of observed variables to innovations in $\boldsymbol{F}_{it}$. Then in the unrestricted specification, the Jacobian of moment conditions (either in levels or differences or both) based on lags of inputs or revenue or their differences does not have full rank.

Proof: see appendix C.

This proposition demonstrates that the rank of Jacobian for the moment conditions is smaller than the number of parameters to be estimated in the unrestricted specification and, hence, the model is not identified. Note that the problem is not in the weak correlation of lags of variables with their current values (which is the point addressed by using level moment conditions). The reduced rank problem arises because profit maximization imposes restrictions on how inputs and outputs comove over time so that some moments are collinear. Of course, the probability of finding a reduced rank is small because various misspecifications can ensure the full rank. Yet the estimator is likely to have a flat density. Furthermore, I show in Appendix B that, to a first order approximation, BB can be poorly identified even when it is costly to adjust inputs. It is critical to use restricted specification to attenuate the problem of weak identification. Mavroeidis (2004) notes a similar problem with using lags as instruments in estimation of rational-expectations macroeconomic models.

Overall, the BB estimator in the production-function context can suffer from weak identification of parameters beyond what has been known before.[19] Note that Propositions 4 and 5 do not show poor identification of system GMM rather they show that poor identification can be a serious problem when the estimator is applied to estimating production functions of optimizing firms.

4.3. INVERSION ESTIMATORS

In this section I consider inversion estimators that use demands for inputs, investment or other observable choices of firms to construct a proxy for firm's productivity and condition inputs in the production function on the proxy. A typical regression in this control-function approach is

$$y_{it} = \eta z_{it} + \lambda \tilde{a}_{it} + \vartheta_{it} = \eta z_{it} + \lambda \tilde{a}_{it} + error, \qquad (17)$$

where $\tilde{a}$ is the proxy for the productivity of a firm. The critical assumption of these estimators is that the mapping (inversion function) from observed characteristics to productivity or its proxy is non-stochastic. I focus on the Levinsohn-Petrin (2003, henceforth LP) estimator but my conclusions are also relevant to similar estimators (e.g., Olley and Pakes 1996, Pavcnik 2002).

Following LP, consider the Cobb-Douglas revenue function with capital, labor and material inputs, that is, $Y_{it} = \exp(a_{it}) K_{it}^{\beta_K} L_{it}^{\beta_L} M_{it}^{\beta_M}$ where the productivity shock a_{it} is an AR(1) process: $a_{it} = \rho_a a_{i,t-1} + \upsilon_{it}^a$ and $\upsilon_{it}^a \sim iid(0, \sigma_{\upsilon a}^2)$. In the notation of LP, $\omega_{it} = a_{it}$ and $\xi_{it} = \upsilon_{it}^a$ and, for convenience, define $\tau_{it} = E(\omega_{it} \mid \omega_{i,t-1}) = \rho_a a_{i,t-1}$. Capital is chosen in the beginning of period t when ξ_{it} is not observed but τ_{it} and factor prices are observed. Labor and materials are chosen when ξ_{it} is known, that is, variable inputs can be adjusted when more information is available.[20] I denote (log) factor prices for capital, labor and materials with r_{it}, w_{it}, and p_{it}^M. Factors are supplied in perfectly competitive markets. There is no measurement error. The rest of the problem is unchanged and the estimated production (revenue) function is

[19] The BB estimator can be identified from nonlinearities in decisions rules captured by second-order effects. In addition, one may expect a better performance of the BB estimator if shocks to factor prices have higher orders of correlation than shocks to productivity. For example, factor prices with AR(2) structure are sufficient to guarantee identification of the BB estimator if productivity is AR(1). However, if the roots (other than the largest root) of the lag polynomial for factor prices are small, the BB moments can be almost collinear in finite samples and the estimator can behave erratically. Furthermore, there is no a priori reason to believe that wage shocks have a higher order of autocorrelation than productivity shocks. Likewise, identification from second-order effects can be fragile. BB can be identified if it is costly to adjust *all* inputs.

[20] Note that, in contrast to inversion estimators, the covariance estimator does not depend on timing assumptions.

$$y_{it} = \beta_K k_{it} + \beta_L l_{it} + \beta_M m_{it} + \omega_{it}, \tag{18}$$

where $\eta = \beta_K + \beta_L + \beta_M$ is returns to scale in the revenue function.

The idea of the LP estimator is to invert demands for capital and materials to infer productivity shocks ω_{it} and then use the estimated productivity shock as a regressor in the production function—that is, condition (18) on ω_{it}. The problem, however, is in the poor quality of the estimates of the productivity shocks.

Note from profit maximization that the observed variables k_{it}, l_{it}, m_{it} and y_{it} can be expressed in terms of unobserved variables r_{it}, w_{it}, p_{it}^M, τ_{it}, and ξ_{it}:

$$(\eta-1)k_{it} = (1+\beta_K-\eta)r_{it} + \beta_L w_{it} + \beta_M p_{it}^M - \tau_{it}, \tag{19}$$

$$(\eta-1)l_{it} = \beta_K r_{it} + (1+\beta_L-\eta)w_{it} + \beta_M p_{it}^M - \tau_{it} - (\eta-1)(\eta-1-\beta_K)^{-1}\xi_{it}, \tag{20}$$

$$(\eta-1)m_{it} = \beta_K r_{it} + \beta_M w_{it} + (1+\beta_M-\eta) p_{it}^M - \tau_{it} - (\eta-1)(\eta-1-\beta_K)^{-1}\xi_{it}, \tag{21}$$

$$(\eta-1)y_{it} = \beta_K r_{it} + \beta_L w_{it} + \beta_M p_{it}^M - \tau_{it} - (\eta-1)(\eta-1-\beta_K)^{-1}\xi_{it}. \tag{22}$$

It is straightforward to invert factor demands to firm's productivity $\omega_{it} = \tau_{it} + \xi_{it}$:

$$\tau_{it} = (1-\eta)k_{it} + (1+\beta_K-\eta)r_{it} + \beta_L w_{it} + \beta_M p_{it}^M, \tag{23}$$

$$\xi_{it} = -(1+\beta_K-\eta)(k_{it} - m_{it} + r_{it} - p_{it}^M). \tag{24}$$

There is a one-to-one non-stochastic mapping between factor demands and productivity shocks if and only if factor prices are the same across firms. But if factor prices are the same across firms then labor and materials are collinear. To see this point, suppose that factor prices are the same across firms in any given period t. Because inversion of factor demands is indexed by time, one can conveniently set r_{it}=r_t=0, w_{it}=w_t=0, and $p_{it}^M = p_t^M = 0$. Clearly, this leads to $m_{it} = l_{it} = \frac{-1}{\eta-1}\tau_{it} + \frac{-1}{\eta-1-\beta_K}\xi_{it}$ and, thus, m_{it} and l_{it} are collinear. Even if the responses of l_{it} and m_{it} to τ_{it} and ξ_{it} are different (e.g., factor supply curves for labor and materials have different slopes), there is no unexplained variation in l_{it} after it is conditioned on m_{it} and k_{it}: $l_{it} - E(l_{it} \mid k_{it}, m_{it}) = l_{it} - E(l_{it} \mid \tau_{it}, \xi_{it}) = l_{it} - l_{it} = 0$. Put differently, once (18) is conditioned on ω_{it} there is no variation in labor/materials ratio and coefficients β_L, β_M are not identified. On the other hand, if factor prices are not the same across firms, then the assumption of non-stochastic inversion function is violated. Therefore, inversion of factor demands and conditioning (18) on estimated productivity shocks are internally inconsistent. In applications, identification of LP must come from misspecification of the model. This point is raised by Basu (1999) and further discussed in Ackerberg and Caves (2003) and Bond and Soderbom (2005).

What happens if volatility in factor prices is ignored? After all, LP indeed moves the estimates in the direction predicted by the theory, e.g., $\beta_L^{OLS} > \beta_L^{LP}$ and $\beta_K^{OLS} < \beta_K^{LP}$. To understand why LP can improve upon OLS, observe that the control function that combines k_{it}, m_{it} and ω_{it} in (18) is

$$\zeta(k_{it}, m_{it}) = \beta_K k_{it} + \beta_M m_{it} + \omega_{it} = (1 + \beta_M + \beta_K - \eta)(m_{it} + p_{it}^M) + \beta_L w_{it}, \tag{25}$$

which is correlated with prices w_{it} and p_{it}^M. What are the consequences? Consider a simple case of one input with perfectly elastic supply and $\tilde{a}_{it} = a_{it} + \chi w_{it}$ as a control function in (17) where $\rho(\tilde{a}_{it}, w_{it}) \neq 0$ if $\chi \neq 0$. Using projection methods, one can find

$$p\lim \hat{\eta}'_{OLS} = \frac{\operatorname{cov}(y_{it}, z_{it})\operatorname{var}(\tilde{a}_{it}) - \operatorname{cov}(y_{it}, \tilde{a}_{it})\operatorname{cov}(z_{it}, \tilde{a}_{it})}{\operatorname{var}(\tilde{a}_{it})\operatorname{var}(z_{it}) - \operatorname{cov}(z_{it}, \tilde{a}_{it})^2} = \eta + (1-\eta)\frac{\chi}{1+\chi}.$$

Clearly, the estimated coefficient is inconsistent unless $\chi = 0$. From (25), it is likely that χ is positive and $\hat{\eta}'_{OLS}$ is biased upward. The performance of LP depends critically on the parameter χ. Specifically, as χ increases, LP converges to OLS.[21] However, for small χ, LP is likely to have large standard errors since variation of z_{it} condition on a_{it} is small. (To reiterate, if χ =0, LP is not identified.) Using nonparametric techniques or polynomials does not resolve the misspecification in (25) and the subsequent identification problem in (18) because identification of LP does not depend on the functional forms.

Measurement errors present another problem in the inversion estimators because the assumption of non-stochastic inversion of observable choices does not hold and upward biases are likely to arise. More generally, conditioning on a proxy variable contaminated with measurement error leads to inconsistent estimates. To get intuition, consider a simple case of one input and $\tilde{a}_{it} = a_{it} + \zeta_{it}$ as a control function in (17), where $\zeta_{it} \sim iid(0, \sigma_\zeta^2)$ is a classical measurement error. It follows that

$$p\lim \hat{\eta}''_{OLS} = \frac{\operatorname{cov}(y_{it}, z_{it})\operatorname{var}(\tilde{a}_{it}) - \operatorname{cov}(y_{it}, \tilde{a}_{it})\operatorname{cov}(z_{it}, \tilde{a}_{it})}{\operatorname{var}(\tilde{a}_{it})\operatorname{var}(z_{it}) - \operatorname{cov}(z_{it}, \tilde{a}_{it})^2} = \eta + (1-\eta)\frac{\sigma_a^2 \sigma_\zeta^2}{\sigma_a^2 \sigma_w^2 + (\sigma_a^2 + \sigma_w^2)\sigma_\zeta^2}.$$

[21] If χ is very large, the OLS bias $(1-\eta)(\sigma_a^2/\sigma_a^2)/(1+\sigma_a^2/\sigma_a^2)$ can be smaller than the LP bias $(1-\eta)\chi/(1+\chi)$.

Clearly, this estimate is not consistent unless $\sigma_\zeta^2 = 0$. Intuitively, because z_{it} is correlated with a_{it}, the attenuation bias in the estimate λ translates into upward bias in the estimate of η. Note that the bias in $\hat{\eta}''_{OLS}$ is strictly increasing in σ_ζ^2 and, as informativeness of $\tilde{a}_{it}$ falls (i.e., $\sigma_\zeta^2 \to \infty$), the probability limit of $\hat{\eta}''_{OLS}$ converges to the probability limit of $\hat{\eta}_{OLS}$. Thus, measurement error in the productivity proxy leads to inconsistent estimates of η although the bias is smaller than in the case of OLS estimates.

Overall, LP estimates of returns to scale are likely to be biased upward, although the bias is smaller than in OLS. The same problems can arise in other inversion-based estimators (e.g., Olley-Pakes 1996, Pavcnik 2002) because the dispersion of factor prices across firms does not allow non-stochastic inversion of firm's observed choices into firm's unobserved productivity. Inversion estimators can be a tenuous solution to the transmission bias problem because they ignore the variation in input mixes and/or measurement errors in inputs.

5. MONTE CARLO EXPERIMENTS

To verify my conclusions and evaluate the performance of the COV estimator, I run a series of Monte Carlo experiments. In each of these experiments, I draw factor prices, productivity and other shocks from the normal distribution and for given realizations of the shocks I compute profit maximizing choices of revenue and inputs. Starting values of shocks are drawn from the corresponding unconditional distributions. For each replication, I generate a panel of 1,000 firms observed for 10 periods, which is close to typical sizes in applied work. I feed the generated data into various estimators and compute the estimates of the parameters for a given production function. I repeat the procedure 1,000 times and report median bias, standard deviation and root mean squared error (MSE) for each of the considered estimators.

In all experiments, returns to scale is $\gamma = 1.1$, which is consistent with the estimates of RTS from reports compiled by engineers (e.g., Pratten 1988), and the markup is $\mu = 2$. I choose a large markup to contrast the performance of the estimators. Returns to scale in the revenue function is $\eta = \gamma / \mu = 0.55$. Factors are supplied in perfectly competitive markets unless other is specified.

I consider the following estimators: OLS, fixed effects (FE), Schmidt's instrumental variables (SIV), Blundell-Bond (BB), COV and, where possible, Levinsohn-Petrin (LP). I use

STATA's commands **xtabond2** and **levpet** for the BB and LP estimators, respectively. Schmidt's (1988) IV estimator uses (if necessary leads or lags of) input ratios as instruments for inputs. COV is estimated by first-difference MLE. I redesign COV estimator for each experiment, that is, I impose restrictions that are relevant for the given data generating process.

5.1. *ONE-INPUT/ONE-OUTPUT*

The data generating process (DGP) for this set of experiments is given in (12)-(15). I start with the simplest calibration that allows no measurement error (Parameterization A, Table 2). Proposition 2 ensures unique global identification of the COV estimator. SIV with $(y_{it} - z_{it})$ as the instrument for z_{it} is also consistent. Note, however, that COV is overidentified while SIV is exactly identified and thus SIV has larger variance than COV.[22] OLS and FE have a predictably large bias in the estimated returns to scale. Although BB has a smaller bias than OLS, the reduction in the bias is small and the standard deviation of the estimates increases substantially. Figure 1 presents the kernel densities of the estimates. In agreement with my theoretical predictions, the density of the BB estimator is essentially flat, which is typical for all experiments and parameterizations that I consider.

Next, I add measurement error to y and z (Parameterization B, Table 2). Again Proposition 2 guarantees global identification of the COV estimator. SIV with $(y_{i,t-1} - z_{i,t-1})$ as the instrument is consistent but it has standard deviation larger than that of COV. BB is considerably worse than FE in terms of MSE. Even OLS has a smaller MSE than BB. The somewhat better performance of OLS and FE can be explained by the fact that the measurement error attenuation (downward bias) partially offsets the upward transmission bias. This is particularly important for FE because the signal to noise ratio for FE falls more than that for OLS (see Griliches and Hausman 1986).

In the next experiment, I add serially correlated measurement error to the input x (Parameterization C, Table 2). In particular, I assume that the measurement error is $\varepsilon_{it}^{z} = \rho_z \varepsilon_{i,t-1}^{z} + e_{it}$ with $\rho_x = 0.8, \sigma_e^2 = 1$. Propositions 3, 4 and 7 formally prove that COV is globally identified. Note that SIV is not consistent because the input/output ratio is correlated with the measurement error and, consequently, the SIV's instrument is correlated with the error

[22] In fact, SIV does not have even first moments because it is exactly identified (Kinal 1980). It is an easy extension to make SIV overidentified by using leads or lags of input ratios wherever appropriate.

term in the production function at all leads and lags. Serial correlation of the measurement error deteriorates the signal to noise ratio and the attenuation bias becomes stronger. This further offsets the transmission bias in OLS and FE estimates. In contrast to serially uncorrelated measurement errors, the bias in the BB estimator is reduced but the standard deviation increases because the production function is quasi-differenced twice.

Finally I consider the case when the factor price and productivity shocks are positively correlated. Specifically, I set $\rho(a_{it}, w_{it}) = 0.7$. This correlation invalidates the SIV estimator because any lead/lag of $(y_{it} - z_{it})$ is correlated with the residual in the production function. To highlight the consequences of the correlation, I assume no measurement errors. Calibration and results are presented in Panel D, Table 2. Because $\rho(w_{it}, a_{it}) \neq 0$, SIV has a very large downward bias so that the estimate of returns to scale is negative. Proposition 2 guarantees global identification of the COV estimator. COV is the only consistent estimator. Note that the bias in OLS, FE and BB estimates of RTS increases considerably because there is less exogenous variation in factor prices. FE is more biased than BB but FE dominates BB in terms of MSE.

5.2. MULTI-INPUT/ONE-OUTPUT

In this section I consider a more realistic setup with multiple inputs (capital, labor and materials) as in Section 4.3. The data generating process is given by (19)-(22) and the estimated production function is (18). I set $\beta_K = 0.1\eta, \beta_L = 0.2\eta, \beta_M = 0.7\eta$. I assume diagonal Ω and Π, i.e., factor price and productivity are uncorrelated. Because capital is predetermined at time *t*, the appropriate instrument for capital in SIV is $(y_{i,t+1} - k_{i,t+1}) = -(\eta - 1 - \beta_K)^{-1} \upsilon^a_{i,t+1} + r_{it}$ that is uncorrelated with $a_{i,t-1}$ and υ^a_{it}.[23]

In the first experiment, I consider the case with no measurement error. Panel A, Table 3 summarizes the calibration and reports the results. COV has the smallest median bias and MSE. SIV has no bias in estimated returns to scale in the revenue function η but the variance of the estimate is large (recall that SIV is exactly identified). The LP estimator does better than OLS but LP still has a sizable upward bias, which is consistent with my theoretical predictions. Furthermore, computationally simpler FE has performance very close to that of the LP estimator.

[23] Timing of shocks modifies the moments used in the BB estimator. However, the fact that output, labor and materials are determined simultaneously and the response of labor, materials and revenue to υ^a is the identical, leads to the reduced rank problem for the BB estimator (see proof of Proposition 5).

The BB estimator has a large negative bias in the coefficient on materials and the largest upward bias on the coefficient on labor. Nonetheless, BB has a relatively small bias in the estimated returns to scale. I plot the kernel density of the estimators in Figure 2. Observe that the density of the LP estimator almost coincides with FE's density. Also note the flat density of the BB estimator.

To show the importance of the small profit share for the estimate of the bias, I vary the demand elasticity so that profit share ranges from 50% to 0%. Figure 3 plots the bias as a function of the profit share. Note that BB, SIV, and LP reduce the bias relative to OLS but as profit share falls these estimators yield only a minor reduction in the bias. Interestingly, LP only marginally improves upon FE. Given that LP and BB tend to have larger variance than OLS, it is not clear if popular solutions to the transmission bias are better in terms of MSE than the OLS estimate.

In the next experiment, I add a small measurement error to inputs and revenue to assess the sensitivity of BB and inversion-based LP to measurement errors. Calibration and results are reported in Panel B, Table 3. Predictably, the attenuation bias partially offsets the transmission bias and, thus, the estimates of RTS are less biased than in the absence of measurement errors. Nonetheless, BB has an increased bias because one has to take more distant lags in the moment conditions. This greatly deteriorates the performance of the estimator. Although the LP estimator has a smaller bias in the estimated returns to scale η, the upward bias in the coefficient on materials is reallocated to the upward bias in the coefficients on capital and labor. Overall, LP is very similar to FE. Only, SIV and COV yield consistent estimates in this experiment.

Next I examine the case with an upward-sloping labor supply curve. I set the elasticity of the labor cost to $\phi_L = 1.5$ and I assume no measurement error. Importantly, although the base wage log(W_{it}) is uncorrelated with productivity a_{it}, the log of wage $W_{it} L_{it}^{\phi-1}$ is correlated with a_{it}. Since the log wage is correlated with a_{it}, SIV is not consistent. Note that the OLS, BB and FE estimates exceed unity although the true returns to scale in the revenue function is 0.55. Only COV estimate the parameters consistently.

In the next series of experiments, I assume quadratic costs of adjustment for capital and keep the rest of the assumptions. In brief, the firm solves the following dynamic problem:

$$E_0 \sum_{t=0}^{\infty} (P_{it} Q_{it} - R_{it} I_{it} - W_{it} L_{it} - P_{it}^M M_{it} - \tfrac{1}{2} \psi (I_{it} / K_{i,t-1} - \delta)^2 K_{i,t-1}) \rightarrow \max$$

$$s.t. \quad P_{it} Q_{it} = A_{it} K_{i,t-1}^{\beta_K} L_{it}^{\beta_L} M_{it}^{\beta_M}, K_{it} = (1-\delta) K_{i,t-1} + I_{it}, W_{it} = W_{it}^0 L_{it}^{\phi-1}, \tag{26}$$

$$r_{it} = \rho_r r_{i,t-1} + \upsilon_{it}^r, w_{it}^0 = \rho_w w_{i,t-1}^0 + \upsilon_{it}^w, p_{it}^M = \rho_{p^M} p_{i,t-1}^M + \upsilon_{it}^M, a = \rho_a a_{i,t-1} + \upsilon_{it}^a$$

where I is investment, ψ is the adjustment cost parameter, small letter denote logs of the respective variables. In all simulations I set $\psi = 6$, which is consistent with the available estimates of adjustment costs (e.g., Gordon 1992), and estimate with other parameters of the model. I log-linearize the first-order conditions and constraints. Because the analytical solution to the above problem is highly complicated, it is hard to establish that the covariance estimator is uniquely globally identified. However, since the numeric solution can be readily written in the state-space form (see Appendix A, section 7.3), it is straightforward to establish local identification of the parameters by checking the rank of the Jacobian.

Using the log-linearized solution to the model, I generate artificial data sets and feed them into various estimators. Table 4 presents the results for the cases with perfectly and imperfectly elastic factor supply curves and with/without measurement errors. In the baseline experiment with perfect competition in factor markets ($\phi = 1$) and no measurement errors (parameterization A), OLS, FE and BB estimates are biased so much that the estimated returns to scale are increasing (recall that the true returns to scale in the revenue function is equal to 0.55). Consistent with the argument in section 4.2, BB estimates have large standard errors, which indicate poor identification of the estimator. Although LP estimator has a smaller bias than other estimators, the size of the bias remains very large. SIV produces implausible estimates because the shadow price of capital is correlated with technology and, hence, no lead or lag of output to capital ratio is a valid instrument for the level of the capital stock. This correlation of shadow price of capital and technology is the key to understanding why the conventional estimators yield increasing returns to scale even when true returns are well below unity. Because of the attenuation bias, adding measurement error (parameterization B) reduces the bias in the estimated returns to scale. In the case with an upward-sloping labor supply curve (parameterization C, $\phi = 1.5$) the bias tends to increase in the estimate of β_L and decrease in the estimate of β_K. Nonetheless, because β_L and β_K have a small contribution to the returns to scale (recall that the elasticity of output with respect to material is 0.7), the bias in the estimate of

RTS barely changes. Note that in all cases, the covariance estimator performs well, although the standard error of the coefficient on capital is somewhat large.

5.3. DISCUSSION

The results of Monte Carlo experiments are in agreement with my theoretical predictions that LP is biased upwards and BB is poorly identified. SIV is extremely sensitive to serially correlated measurement errors and (shadow) factor prices correlated with technology. The experiments show that simpler OLS and FE have performance comparable to that of BB and LP. If the profit share is small, the reduction in the bias from using BB and LP is offset by an increase in the variance of the estimates. Hence, in empirically plausible settings with small profit shares, it is useful to compare estimates from sophisticated techniques with OLS estimates.

Importantly, the Monte Carlo experiments suggest that the puzzling estimates of returns to scale in the revenue function can arise because statistical estimators fail to provide consistent estimates of returns to scale. In the next section, I contrast the estimates of competing techniques when applied to real data.

6. APPLICATION

In this section, I apply the COV estimator to a well-known data set of Chilean manufacturing plants. Lui (1991, 1993), Lui and Tybout (1996), Pavcnik (2002) and Petrin and Levinsohn (2005) describe the data in detail. To illustrate the estimator, I focus on SIC 3240 industry (Manufacture of footware).[24] The annual data spans from 1982 to 1996. Descriptive statistics for logs of real value added, real capital stock and labor are presented in Table 5 and 6

I assume that inverse demand function is given by $P_{it} = GQ_{it}^{-1/\sigma}$, where σ is the demand elasticity, the markup is $\mu = \sigma/(\sigma-1)$, G is a demand shifter. The production function is described by $Q_{it} = A_{it}^{\mu} \min\{M_{it}, cK_{it}^{\alpha_K} L_{it}^{\alpha_L}\}$ where Q_{it} is output in physical units, M_{it} is the input of materials, K_{it} is capital, L_{it} is the number of employees, A_{it} is the level of Hicks-neutral technology (A_{it} to the power of μ is a normalization), c is a constant of proportionality. This functional form imposes zero substitution between materials and combined capital/labor inputs. Since at the optimum no resources are wasted, $M_{it} = cK_{it}^{\alpha_K} L_{it}^{\alpha_L}$ and, hence, the profit function is

[24] I am grateful to Jim Levinsohn for providing me with the data.

given by $\pi_{it} = P_{it}Q_{it} - p_{it}^M M_{it} - R_{it}K_{it} - W_{it}L_{it} = VA_{it} - R_{it}K_{it} - W_{it}L_{it}$, where VA_{it} is the value added, R_{it} and W_{it} are the cost of capital and labor for firm i at time t.[25] In the data, the share of materials in total cost is 0.66.

I assume that capital is supplied in perfectly competitive markets. The slope of the labor supply curve is a free parameter. In particular, I assume that the wage function is given by $W_{it}(L) = W_{it} L_{it}^{\phi-1}$ so that the wage bill is $W_{it}(L_{it})L_{it} = W_{it} L_{it}^{\phi}$. I further assume that capital, labor, and revenue are chosen simultaneously. I allow serially correlated errors in all observed variables, which are capital, labor and revenue.[26] Unobserved technology and factor prices are serially correlated and there could be feedback from technology to factor prices and vice versa. I assume that innovations to technology and factor prices are uncorrelated. In summary, the estimated model is

$$y_{it}^* - k_{it}^* = r_{it}, \tag{27}$$
$$y_{it}^* - l_{it}^* = w_{it} + (\phi - 1)l_{it}^*, \tag{28}$$
$$y_{it}^* = a_{it} + \beta_K k_{it}^* + \beta_L l_{it}^*, \tag{29}$$
$$a_{it} = \rho_{aa} a_{i,t-1} + \rho_{aw} w_{i,t-1} + \rho_{ar} r_{i,t-1} + \upsilon_{it}^a, \tag{30}$$
$$w_{it}^0 = \rho_{wa} a_{i,t-1} + \rho_{ww} w_{i,t-1}^0 + \rho_{wr} r_{i,t-1} + \upsilon_{it}^w, \tag{31}$$
$$r_{it} = \rho_{ra} a_{i,t-1} + \rho_{rw} w_{i,t-1} + \rho_{rr} r_{i,t-1} + \upsilon_{it}^r, \tag{32}$$
$$y_{it} = y_{it}^* + u_{it}^y, \tag{33}$$
$$k_{it} = k_{it}^* + u_{it}^k, \tag{34}$$
$$l_{it} = l_{it}^* + u_{it}^l, \tag{35}$$
$$u_{it}^y = \rho_y u_{i,t-1}^y + \varepsilon_{it}^y, \tag{36}$$
$$u_{it}^k = \rho_k u_{i,t-1}^k + \varepsilon_{it}^k, \tag{37}$$
$$u_{it}^l = \rho_l u_{i,t-1}^l + \varepsilon_{it}^l, \tag{38}$$

[25] This trick helps to circumvent the problem of measuring the quantity of the materials input. Note that in the vast majority of cases the researcher knows only the nominal spending on materials and the quantity of the material input is obtained by deflating the nominal spending with industry-level material price index. Since the mix of intermediate inputs varies across firms and the price index is the same for all firms in any given period, the computed quantity of the material input can be poorly correlated with the true quantity of the material input. In the case of the Cobb-Douglass production function, the nominal spending on materials is proportional to revenue and, hence, including the deflated expenditures on materials should yield perfect collinearity. Stochastic errors (e.g., optimization errors, measurement errors) can break the collinearity but the coefficient is still likely to be close to unity, which is often the case in applications.

[26] In the case of revenue, errors can be interpreted as innovations to technology or demand that are not transmitted to input choices (see Zellner, Kmenta and Dreze 1966).

where small letters denote logs of the respective variables with $y_{it} = \ln VA_{it}$, stars denote true values, $\{\upsilon_{it}^a, \upsilon_{it}^w, \upsilon_{it}^r, \varepsilon_{it}^y, \varepsilon_{it}^k, \varepsilon_{it}^l\}$ are uncorrelated i.i.d. innovations. Parameters of interest are β_K, β_L and returns to scale in the value-added function $\eta = \beta_K + \beta_L$. Equations (27) and (28) are the first order conditions for capital and labor. Equation (29) is the value-added production function. Equations (30)-(32) describe the evolution of structural shocks to productivity and factor prices. Measurement equations are collected in (33)-(35). Dynamics of the measurement errors is in equations (36)-(38). The equations can succinctly rewritten in the matrix form that corresponds to the state space representation in (8)-(9):

$$\boldsymbol{X}_{it} \equiv \begin{bmatrix} y_{it} \\ k_{it} \\ l_{it} \end{bmatrix} = \left[\begin{array}{ccc|ccc} \frac{-\phi}{\beta_L+\beta_K\phi-\phi} & \frac{\beta_L}{\beta_L+\beta_K\phi-\phi} & \frac{\beta_K}{\beta_L+\beta_K\phi-\phi} & 1 & 0 & 0 \\ \frac{-\phi}{\beta_L+\beta_K\phi-\phi} & \frac{\beta_L}{\beta_L+\beta_K\phi-\phi} & \frac{\beta_L-\phi}{\beta_L+\beta_K\phi-\phi} & 0 & 1 & 0 \\ \frac{-1}{\beta_L+\beta_K\phi-\phi} & \frac{1-\beta_K}{\beta_L+\beta_K\phi-\phi} & \frac{\beta_L}{\beta_L+\beta_K\phi-\phi} & 0 & 0 & 1 \end{array}\right] \begin{bmatrix} a_{it} \\ w_{it} \\ r_{it} \\ u_{it}^y \\ u_{it}^k \\ u_{it}^l \end{bmatrix} = \left[\Lambda \;\middle|\; I_3\right] \boldsymbol{F}_{it},$$

$$\boldsymbol{F}_{it} \equiv \begin{bmatrix} a_{it} \\ w_{it} \\ r_{it} \\ u_{it}^y \\ u_{it}^k \\ u_{it}^l \end{bmatrix} = \left[\begin{array}{ccc|ccc} \rho_{aa} & \rho_{aw} & \rho_{ar} & 0 & 0 & 0 \\ \rho_{wa} & \rho_{ww} & \rho_{wr} & 0 & 0 & 0 \\ \rho_{ra} & \rho_{rw} & \rho_{rr} & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & \rho_y & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_k & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_l \end{array}\right] \begin{bmatrix} a_{i,t-1} \\ w_{i,t-1} \\ r_{i,t-1} \\ u_{i,t-1}^y \\ u_{i,t-1}^k \\ u_{i,t-1}^l \end{bmatrix} + \begin{bmatrix} \upsilon_{it}^a \\ \upsilon_{it}^w \\ \upsilon_{it}^r \\ \varepsilon_{it}^y \\ \varepsilon_{it}^k \\ \varepsilon_{it}^l \end{bmatrix} = \left[\begin{array}{c|c} \Pi & 0 \\ \hline 0 & R \end{array}\right] \boldsymbol{F}_{it} + \left[\begin{array}{c} \boldsymbol{\upsilon}_{it} \\ \hline \boldsymbol{\varepsilon}_{it} \end{array}\right].$$

The model has 21 parameters:

$$\theta = \{\beta_K, \beta_L, \phi, \rho_{aa}, \rho_{aw}, \rho_{ar}, \rho_{wa}, \rho_{ww}, \rho_{wr}, \rho_{ra}, \rho_{rw}, \rho_{rr}, \sigma_{\upsilon a}, \sigma_{\upsilon w}, \sigma_{\upsilon r}, \sigma_{\varepsilon y}, \sigma_{\varepsilon k}, \sigma_{\varepsilon l}, \rho_y, \rho_k, \rho_l\},$$

where $\sigma_{\upsilon a}, \sigma_{\upsilon w}, \sigma_{\upsilon r}$ are standard deviations of innovations to productivity (υ_{it}^a), wages (υ_{it}^w) and interest rate (υ_{it}^r), $\sigma_{\varepsilon y}, \sigma_{\varepsilon k}, \sigma_{\varepsilon l}$ are innovations to measurement errors in value added (ε_{it}^y), capital stock (ε_{it}^k), and labor input (ε_{it}^l). Although it is straightforward to verify that the model is locally identified almost everywhere, global identification is hard to show algebraically. To verify that there are no other solutions, I experiment with different starting values. I use MLE given in (11) to estimate the model.

I report the estimation results in Table 7, column 1. Since the data is not normally distributed, I bootstrap the estimates to correct the bias and improve the confidence intervals.[27] Using bootstrapped critical values, I do not reject the model at any conventional significance level (p-value=0.4). To contrast the results, I estimate production function $v_{it} = \beta_K k_{it} + \beta_L l_{it} + \xi_{it}$ by OLS, FE, LP, and BB estimators and report these results in columns 2 to 7 in Table 7. I report two versions of the BB estimator: quasi-differenced (column 6) and twice-quasi-differenced (column 7).

BB, LP and FE estimators yield RTS in a 0.62 to 0.9 range. These estimates suggest a very large 10-38% profit share in value added if factor markets are perfectly competitive. In contrast, the observed (*accounting*) profit share in value added is 2%.[28] Also observe that consistent with our theoretical results and Monte Carlo experiments, the BB estimator has very large standard errors and LP estimates are close to FE estimates. On the other hand, the OLS estimate (RTS=1.30) is inconsistent with profit maximization if factor markets are perfectly competitive. In addition, OLS estimate of β_L imply increasing returns in labor. The SIV estimator yields implausibly large returns to scale. This cacophony in the estimates can be reconciled by the COV estimates.

First, note that the COV estimates returns to scale in the revenue function to be 1.17, which is in line with our argument that the bias in the OLS estimate of returns to scale is likely to be relatively small. Second, the estimate of ϕ is greater than unity and, thus, the firm faces an upward-sloping labor supply curve. Since OLS is biased to ϕ, the OLS estimate of RTS is greater than COV estimate of RTS. Third, I find relatively large measurement errors. These errors tend to attenuate the estimates toward zero, especially when estimates are from within variation. This can explain why FE, BB, and LP produce low returns to scale. Note that the small coefficient on capital in BB is consistent with strong downward bias in β_K in my Monte Carlo experiments with serially correlated measurement errors. Finally, since the SIV estimator uses output to input ratios as instruments and measurement error is present in inputs and factor prices

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[27] I use non-parametric bootstrap with resampling firms. See Horowitz (1998) for the discussion of bootstrap for covariance structures.

[28] The profit share is computed as the ratio of aggregate gross profit to aggregate value added. Although it is hard to sign the bias of the accounting profit as a measure of economic profit, the small magnitude of the profit share consistent with the discussion in section 2. Alternative definitions of the profit share are in the range of 0.2% to 2.5%.

are correlated with technology, the instruments used in the SIV are correlated with the error term in the production function so that the estimates of β_L and β_K behave wildly.

Increasing returns in the revenue function do not contradict profit maximization because the labor supply curve is upward sloping. Specifically, the elasticity of the labor cost $\phi = 1.42$ (i.e., the wage premium is 42%) is generally in agreement with the estimates from previous studies. For example, Shapiro (1986, 1996) and Bils (1987) estimate from the aggregate US data that the shift premium is about 25-40%. Manning (2004) observes that a plausible elasticity of the labor supply is between 2 and 10. In the present case, the implied elasticity of the labor supply curve $1/(\phi - 1) = 2.4$ falls nicely in this interval.

According to (A.3) in Appendix A, I find that the implied elasticity of the cost for capital and labor is 1.21 and elasticity of the cost for all inputs is 1.07. Using (A.4) in Appendix A to compute the profit share from the COV estimates, I find that the profit share is 1.3%, which is a significant improvement in comparison to other estimators.

Note that variation in factor prices is comparable to variation in productivity a_{it}. Specifically, the point estimates in Table 7 imply that $\sigma(a_{it}) = 0.332$, $\sigma(w_{it}) = 0.513$, $\sigma(r_{it}) = 0.230$. This supports other evidence on the dispersion of prices even in narrowly defined industries. I also conclude that ignoring variation in factor prices across firms can lead to serious identification problems for the inversion-based estimators. Finally, since markup $\mu \geq 1$ and returns to scale in production function $\gamma = \mu\eta \geq \eta$, one can expect sizable increasing returns to scale in production.

7. CONCLUSION

This paper has critical and constructive parts. In the critical part, I demonstrate that under weak assumptions estimates from production function regressions using firm-level data are often inconsistent with profit maximization or imply implausibly large profits. Specifically, I argue that firm-level data limitations lead to estimating returns to scale in the revenue function. On the other hand, I prove that returns to scale in the revenue function cannot be greater than unity as long as the profit share in revenue is non-negative and factor supplies are perfectly elastic. This sharply contrasts with frequent finding that returns to scale in the revenue function at the firm level exceed unity. On the econometric front, I point out that inversion-based estimators and

GMM/IV estimators that use lags of endogenous variables as instruments can be poorly identified so that the estimates of returns to scale can be seriously distorted.

In the constructive part, I show that under weak assumptions the elasticity of the factor supply can reconcile increasing or large decreasing returns in the revenue function and a small non-negative profit share. Furthermore, I argue that simple structural estimators that model the cost and the revenue function simultaneously and treat unobserved heterogeneity in productivity and factor prices symmetrically can resolve many of the problem I identify above. I provide and example and illustrate the strength of the suggested estimator in Monte Carlo simulations and in an empirical application.

The paper has broader implications. First, I argue that the profit share can be used as a robust non-parametric *economic* diagnostic for estimates of returns to scale. Second, although I analyze only one industry, it is clear that variation in product and factor prices at the firm-level is not trivial. This entails important consequences for aggregating firm-level data (and devastating effects on the inversion-based estimators). Specifically, reallocation effects due to heterogeneity in factor prices are likely to be of first-order importance. Furthermore, productivity aggregates measure revenue generating ability in the industry rather than technical efficiency. Third, since it is fairly common to find constant returns to scale in the revenue function at the firm level and markup is not less than unity, returns to scale in production at the firm level can be sizeable. Hence, business cycle and trade models appropriately calibrated can produce qualitatively different results. In addition, the gap between RTS in aggregate data and RTS in firm-level data is smaller than thought before. Fourth, factor supply curves are likely to be upward sloping at the firm level. This means that the cost-weighted composite input does not measure the total input correctly and factor prices can be procyclical irrespective of the source of the shocks.

8. References

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Appendix A: Alternative economic models

8.1. *Multi-input case*

This appendix presents the multi-input analogue for the model considered in Section 2.1. The production function is assumed to be Cobb-Douglas: $Q_{it} = A_{it}^{1/\mu}\prod_{j=1}^{n} L_{j,it}^{\alpha_j}$ where i, t, j index firms, time, and inputs, $\gamma = \sum_{j=1}^{n}\alpha_j$ is returns to scale in production, A_{it} is Hicks-neutral firm-specific productivity, and $L_{j,it}$ is j^{th} input. The inverse demand function is isoelastic $P_{it} = G_{it}\cdot Q_{it}^{-1/\sigma}$ where P_{it} is the price of the good, Q_{it} is the quantity of the good, G is a demand shifter, and σ is the elasticity of demand. The markup is $\mu = \sigma/(\sigma-1)$. Hence, the revenue function is $Y_{it} = P_{it}Q_{it} = G_{it}(A_{it}^{\mu}\prod_{j=1}^{n} L_{j,it}^{\alpha_j})^{1-1/\sigma} = G_{it}A_{it}\prod_{j=1}^{n} L_{j,it}^{\beta_j}$, where $\beta_i = \alpha_i/\mu$, and $\eta = \sum_{j=1}^{n}\beta_j$ is returns to scale in the revenue function. Also note that A_{it} and G_{it} are isomorphic in the revenue function so that the econometrician cannot separate these shocks. Hence, I drop G_{it} from the analysis and concentrate on A_{it} only. The cost for input j is given by $C_j(L_j) = W_{j,it}L_{j,it}^{\phi_j}$ where ϕ_j is the elasticity of the cost of input j. The profit maximization problem is then

$$\pi_{it} \equiv Y_{it} - \sum_{j=1}^{n-1} C_j(L_j) = A_{it}\prod_{j=1}^{n} L_{j,it}^{\beta_j} - \sum_{j=1}^{n-1} W_{j,it}L_{j,it}^{\phi_j} \xrightarrow[L_{1,it},\ldots,L_{n-1,it}]{} \max,$$

I take logs of the first order conditions, suppress uninteresting constants, partial out industry-wide shocks, and get the following expressions for optimal input choices and revenue

$$\begin{bmatrix} -\phi_1 & 0 & 0 & \cdots & 0 & 1 \\ 0 & -\phi_2 & 0 & \cdots & 0 & 1 \\ 0 & 0 & -\phi_3 & \cdots & 0 & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -\phi_{n-1} & 1 \\ -\beta_1 & -\beta_2 & -\beta_3 & \cdots & -\beta_{n-1} & 1 \end{bmatrix} \begin{bmatrix} l_{1,it} \\ \vdots \\ l_{n-1,it} \\ y_{it} \end{bmatrix} = \begin{bmatrix} w_{1,it} \\ \vdots \\ w_{n-1,it} \\ a_{it} \end{bmatrix} \Rightarrow \boldsymbol{X}_{it} \equiv \begin{bmatrix} l_{1,it} \\ \vdots \\ l_{n-1,it} \\ y_{it} \end{bmatrix} = \Lambda \begin{bmatrix} w_{1,it} \\ \vdots \\ w_{n-1,it} \\ a_{it} \end{bmatrix} = \Lambda \boldsymbol{F}_{it},$$

where

$$\Lambda = \frac{1}{\sum_{j=1}^{n-1}\beta_j\psi_{jj} - \prod_{i=1}^{n-1}\phi_i} \begin{bmatrix} \psi_{11} - \sum_{i\neq 1}\beta_i\psi_{1i} & \psi_{12}\beta_2 & \psi_{13}\beta_3 & \cdots & \psi_{1,n-1}\beta_{n-1} & -\psi_{11} \\ \psi_{21}\beta_1 & \psi_{22} - \sum_{i\neq 2}\beta_i\psi_{2i} & \psi_{13}\beta_3 & \cdots & \psi_{1,n-1}\beta_{n-1} & -\psi_{22} \\ \psi_{31}\beta_1 & \psi_{32}\beta_2 & \psi_{33} - \sum_{i\neq 3}\beta_i\psi_{3i} & \cdots & \psi_{1,n-1}\beta_{n-1} & -\psi_{33} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \psi_{n-1,1}\beta_1 & \psi_{n-1,2}\beta_2 & \psi_{13}\beta_3 & \cdots & \psi_{n-1,n-1} - \sum_{i\neq n-1}\beta_i\psi_{n-1,i} & -\psi_{n-1,n-1} \\ \psi_{11}\beta_1 & \psi_{22}\beta_2 & \psi_{33}\beta_3 & \cdots & \psi_{n-1,n-1}\beta_{n-1} & -\prod_{i=1}^{n-1}\phi_i \end{bmatrix}, \tag{A.1}$$

$\psi_{ij} = \prod_{s\neq i,j}^{n-1}\phi_s$ and $\psi_{ij} = 1$ if $s \neq i, j$ is an empty set. Observe that $\det\Lambda = \sum_{j=1}^{n-1}\beta_j\psi_{jj} - \prod_{i=1}^{n-1}\phi_i < 0$ which is the necessary and sufficient condition for the profit function to be concave.

One can use information from the first order conditions to compute the cost shares. Observe that for each input j, the first order condition is $\beta_j Y_{it}/L_{j,it} = \phi_j W_{j,it}L_{j,it}^{\phi_j-1}$. Hence, $C_j(L_j) = W_{j,it}L_{j,it}^{\phi_j} = \beta_j Y_{it}/\phi_j$. If follows that the cost share for input j is given by

$$\omega_j = \frac{C_j(L_j)}{\sum_{h=1}^{n} C_h(L_h)} = \frac{\beta_j/\phi_j}{\sum_{h=1}^{n}\beta_h/\phi_h}. \tag{A.2}$$

The elasticity of the cost with respect to all inputs is

$$\phi = \sum_{h=1}^{n} \omega_h \phi_h = \frac{\sum_{h=1}^{n} \beta_h}{\sum_{h=1}^{n} \beta_h / \phi_h} . \tag{A.3}$$

Using this expression one can find the profit share in terms of cost and revenue elasticities:

$$s_\pi = 1 - \frac{\gamma}{\mu\phi} = 1 - \sum_{h=1}^{n} \beta_h / \phi_h . \tag{A.4}$$

8.2. CONSTANT ELASTICITY OF SUBSTITUTION (CES) PRODUCTION FUNCTION

Consider the CES production function $Q = A^{1/\mu}(\omega_K K^{1/\rho} + \omega_L L^{1/\rho})^{\gamma\rho}$ where $\frac{1}{1-\rho}$ is the elasticity of substitution. In this example, I assume that productivity and factor prices are mutually uncorrelated. Otherwise the structure is the same as in the Cobb-Douglas case. The profit function is given by: $\pi = A(\omega_K K^{1/\rho} + \omega_L L^{1/\rho})^{\gamma\rho/\mu} - RK - WL$. The first order conditions with respect to capital and labor are: $\eta s_K Y / K = R$, $\eta s_L Y / L = W$ where $s_K = \omega_K K^{1/\rho} / (\omega_K K^{1/\rho} + \omega_L L^{1/\rho})$, $s_L = 1 - s_K$. After log-linearizing first-order conditions and the revenue function, one has the following structural equations: $y = \eta s_K k + \eta s_L l + a$, $y - k + \rho s_L (k - l) = r$, $y - l + \rho s_K (l - k) = w$. The reduced form is

$$X = \begin{bmatrix} y \\ k \\ l \end{bmatrix} = \begin{bmatrix} \frac{1}{1-\eta} & -\frac{\eta s_K}{1-\eta} & -\frac{\eta(1-s_K)}{1-\eta} \\ \frac{1}{1-\eta} & -\frac{1-\eta+s_K(\eta-\rho)}{(1-\rho)(1-\eta)} & -\frac{(\eta-\rho)(1-s_K)}{(1-\rho)(1-\eta)} \\ \frac{1}{1-\eta} & -\frac{s_K(\eta-\rho)}{(1-\rho)(1-\eta)} & -\frac{1-\rho-s_K(\eta-\rho)}{(1-\rho)(1-\eta)} \end{bmatrix} \begin{bmatrix} a \\ r \\ w \end{bmatrix}$$

The model has six parameters: $\theta = \{\eta, s_K, \rho, \sigma_a, \sigma_w, \sigma_r\}$. It is straightforward (but tedious) to show that $\nabla_\theta E(XX')$ has full rank and, hence, the model is locally identified almost everywhere.

8.3. RATIONAL EXPECTATIONS

Following Anderson and Moore (1985), one can show that, after log-linearization, rational profit-maximizing producer behavior can be summarized as follows:

$$\boldsymbol{S}_t \equiv \begin{bmatrix} \boldsymbol{G}_t \\ \boldsymbol{H}_t \\ \boldsymbol{Z}_t \end{bmatrix} = \begin{bmatrix} 0 & \Pi_{12} & \Pi_{13} \\ 0 & \Pi_{22} & \Pi_{23} \\ 0 & 0 & \Pi_{33} \end{bmatrix} \begin{bmatrix} \boldsymbol{G}_{t-1} \\ \boldsymbol{H}_{t-1} \\ \boldsymbol{Z}_{t-1} \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} \boldsymbol{v}_t = \Pi \boldsymbol{S}_{t-1} + B\boldsymbol{v}_t \tag{A.5}$$

where $\boldsymbol{G}_t$ is px1 vector of endogenous non-predetermined variables (e.g., materials), $\boldsymbol{H}_t$ is mx1 vector of endogenous predetermined variables (e.g., capital), $\boldsymbol{Z}_t$ is nx1 vector of exogenous variables, $\boldsymbol{v}_t$ is vector of innovations to exogenous variables. The econometrician observes only $\boldsymbol{G}_t$ and $\boldsymbol{H}_t$. The number of shocks is not less than the number of observed variables, i.e., $n \geq m + p$. No assumptions are made about the sources of shocks $\boldsymbol{v}_t$, which can be shocks to adjustment costs, factor prices, productivity, etc. Hence, the setup is very general.

The autocovariance matrix of the observed variables collected in $\boldsymbol{X}_t = [\boldsymbol{G}_t' \quad \boldsymbol{H}_t']' = \Upsilon \boldsymbol{S}_t$ with $\Upsilon \equiv [I_{p+m} \,\vdots\, 0]$ is given by $\tilde{\Gamma}_k \equiv E(\boldsymbol{X}_t \boldsymbol{X}_{t-k}') = \Upsilon \Pi^k \Gamma_0 \Upsilon'$, k=0,1,…, where $\Gamma_0 \equiv E(\boldsymbol{S}_t \boldsymbol{S}_t')$, $vec(\Gamma_0) = (I_{m+p+n} - \Pi \otimes \Pi)^{-1} vec(B\Omega B')$, and $\Omega \equiv E(\boldsymbol{v}_t \boldsymbol{v}_t')$. Given that $\tilde{\Gamma}_k$, k=0,1,…, and matrices Π, B are deterministic one-to-one functions of structural parameters, one can use methods presented in section 3.4 to set up likelihood function for MLE. Specifically, the log-likelihood function for the no-firm-specific-effects and no-measurement-error case is given by $\sum_{i=1}^{N} l(\boldsymbol{X}_i,\theta) = \ln|\Phi_T| + trace\{\hat{\Phi}_T\Phi_T^{-1}\} - \ln|\hat{\Phi}_T| - Tn$ where $\hat{\Phi}_T = \frac{1}{N}\sum_{i=1}^{N}\boldsymbol{X}_i\boldsymbol{X}_i'$, n is the number of observed choices of firms, and

$$\Phi_T \equiv \begin{bmatrix} \tilde{\Gamma}_0 & & & \\ \tilde{\Gamma}_1 & \ddots & & \\ \vdots & \ddots & \ddots & \\ \tilde{\Gamma}_T & \cdots & \tilde{\Gamma}_1 & \tilde{\Gamma}_0 \end{bmatrix} = \begin{bmatrix} \Upsilon\Gamma_0\Upsilon' & & & \\ \Upsilon\Pi\Gamma_0\Upsilon' & \ddots & & \\ \vdots & \ddots & \ddots & \\ \Upsilon\Pi^T\Gamma_0\Upsilon' & \cdots & \Upsilon\Pi\Gamma_0\Upsilon' & \Upsilon\Gamma_0\Upsilon' \end{bmatrix}.$$

The likelihood can be easily extended to cases with measurement errors and firm-specific effects.

Note that model (A.5) is highly nonlinear in structural parameters. Hence, global identification is hard to prove in general. Local identification is easy to verify (numerically) by checking the rank of the relevant Jacobian.

8.4. IDENTIFICATION IN MODELS WITH ADJUSTMENT COSTS

In this section I show that, under certain assumptions, the Blundell-Bond estimator is poorly identified even in the presence of adjustment costs. In the spirit to the results in Section 4.2, poor identification arises because profit maximization imposes restrictions on the dynamic and contemporaneous comovement of inputs and output. The following proposition gives the necessary condition for identification of the BB estimator for any rational expectation model described by (A.5) in Appendix A (Section 9.3).

Proposition 6.

Consider rational profit-maximizing firm characterized by the reduced-form dynamics as in (A.5). Then if $2(m+p)-1>m+n$*, the unconstrained Blundell-Bond estimator is not identified.*
Proof: see appendix C.

Note that this proposition gives only a necessary condition for identification of the Blundell-Bond estimator. Profit maximization can impose further restrictions on the dynamic and contemporaneous correlation between variables so that the estimator is not identified even when the presented *necessary* condition is satisfied. The following proposition provides an important example where BB is not identified although the necessary condition is satisfied.

Proposition 7.

Consider rational profit-maximizing firm characterized by the reduced-form dynamics as in (A.5). Suppose that Π_{33} *is diagonal and that output and one of the inputs are free to adjust contemporaneously in response to shocks. Then the unconstrained Blundell-Bond estimator is not identified if the production function is Cobb-Douglass.*
Proof: see appendix C.

Using the argument of Proposition 4 it is straightforward to show that even when BB is locally identified, there could be several solutions.

9. APPENDIX B: PROOFS

Proof of Proposition 1.

Consider cost minimization problem, which is implied by profit maximization:
$L \equiv (L_1,...,L_s) = \arg\min_L \{\sum_{j=1}^n w_j(L_j) : f(L) = Q\}$, where $w_j(L_j)$ is the cost for input L_j and Q is output. I assume that the total cost is separable in inputs, i.e., $w(L) = \sum_{j=1}^n w_j(L_j)$. If factor markets are competitive, then $w_j(L_j) = w_j L_j$.

The first order condition gives $w_j'(L_j) = \lambda f_j'(L)$ for $j = 1,...,n$ where λ is the Lagrange multiplier and f is the production function. Multiply both sides by L_j for each j sum over j to get

$$\sum_{j=1}^n w_j'(L_j)L_j = \lambda \sum_{j=1}^n f_j'(L)L_j \qquad \Leftrightarrow$$

$$Q \cdot AC(Q) \cdot \sum_{j=1}^n \frac{w_j'(L_j)L_j}{w_j(L_j)} \frac{w_j(L_j)}{TC(Q)} = MC(Q) \cdot \sum_{j=1}^n f_j'(L)L_j \qquad \Leftrightarrow$$

$$\frac{AC(Q)}{MC(Q)}\left(\sum_{j=1}^n \phi_j \omega_j\right) = \frac{1}{Q}\sum_{j=1}^n f_j'(L)L_j \Leftrightarrow \text{(by Euler's theorem) } \frac{AC(Q)}{MC(Q)}\phi = \gamma,$$

where ϕ_j is the elasticity of j^{th} factor price, ω_j is the share of factor i in total cost $TC(Q)$, $\phi = \sum_{j=1}^n \phi_j \omega_j$ is the elasticity of the cost with respect to inputs, $AC(Q)$ and $MC(Q)$ are average and marginal costs. If factor markets are competitive, $\phi_j = 1$ for all j and hence $\phi = 1$. Now observe that profit share is equal to
$s_\pi = \frac{PQ - AC(Q)\cdot Q}{PQ} = 1 - \frac{AC(Q)}{P}$. It follows that $\gamma = \phi\, AC(Q)/MC(Q) = \phi(1-s_\pi)\mu \Leftrightarrow \gamma/\mu\phi = (1-s_\pi)$, where $\mu = P/MC(Q)$ is the markup.

Since marginal revenue (MR) is equal to marginal cost for a profit-maximizing firm, one has $(\partial TR/\partial L_j) = MR(\partial Q/\partial L_j) \Rightarrow \sum_{j=1}^n (\partial TR/\partial L_j)L_j = MC\sum_{j=1}^n (\partial Q/\partial L_j)L_j = (MC/P)\gamma PQ$ and hence

$$\eta = \frac{\sum_{j=1}^n (\partial TR/\partial L_j)L_j}{PQ} = \frac{\gamma}{\mu} \blacksquare$$

Proof of Corollary 1.

Consider cost minimization problem: $L \equiv (L_1,...,L_k,\bar{L}_{k+1},...,\bar{L}_n) = \arg\min_{L_1,...,L_k} \{\sum_{j=1}^n w_j(L_j) : f(L) = Q\}$, where $w_j(L_j)$ is the cost function for input L_j, f is the production function and Q is output and k+1,…,n inputs are fixed. Using the arguments of Proposition 1, one can show that the first order condition with respect to variable $w_j'(L_j) = \lambda f_j'(L)$ for $j = 1,...,k$ (λ is the Lagrange multiplier) yield:

$$\sum_{j=1}^k w_j'(L_j)L_j = \lambda \sum_{j=1}^k f_j'(L)L_j \qquad \Leftrightarrow$$

$$Q \cdot AC(Q) \cdot \frac{\sum_{i=1}^k w_i(L_i)}{TC(Q)} \sum_{j=1}^k \frac{w_j'(L_j)L_j}{w_j(L_j)} \frac{w_j(L_j)}{\sum_{i=1}^k w_i(L_i)} = MC(Q) \cdot \sum_{j=1}^k f_j'(L)L_j \qquad \Leftrightarrow$$

$$\frac{AC(Q)}{MC(Q)} \frac{\sum_{i=1}^k w_i(L_i)}{TC(Q)}\left(\sum_{j=1}^k \phi_j \omega_j\right) = \frac{1}{Q}\sum_{j=1}^k f_j'(L)L_j \Leftrightarrow \text{(by Euler's theorem)}$$

$$\frac{AC(Q)}{MC(Q)}\phi^*\omega^* = \gamma^*,$$

where ω^* is the cost share of variable inputs in total cost, $\phi^* = \sum_{j=1}^{k}\phi_j\omega_j$ is the elasticity of cost with respect to variable inputs, γ^* is returns to scale in production with respect to variable inputs, $AC(Q)$ and $MC(Q)$ are average and marginal costs. Now observe that profit share is equal to $s_\pi^* = 1 - AC(Q)/P$. It follows that

$$\frac{AC(Q)}{MC(Q)}\phi^*\omega^* = \gamma^* \Leftrightarrow \frac{AC(Q)}{P}\frac{P}{MC(Q)}\phi^*\omega^* = \gamma^* \Leftrightarrow (1-s_\pi^*)\mu\phi^*\omega^* = \gamma^*$$

where $\mu = P/MC(Q)$ is the markup.

Since marginal revenue (MR) is equal to marginal cost for a profit-maximizing firm, one has

$$(\partial TR/\partial L_j) = MR(\partial Q/\partial L_j) \Rightarrow \sum_{j=1}^{k}(\partial TR/\partial L_j)L_j = MC\sum_{j=1}^{k}(\partial Q/\partial L_j)L_j = (MC/P)\hat{\gamma}PQ \Rightarrow$$

$$\eta^* = \frac{\sum_{j=1}^{k}(\partial TR/\partial L_j)L_j}{PQ} = \frac{\gamma^*}{\mu},$$

where η^* is returns to scale in the revenue function with respect to variable inputs. By combining the results, one can find: $\eta^* = \gamma^*/\mu = (1-s_\pi^*)\phi^*\omega^*$.■

Proof of Proposition 2

Without loss of generality assume that there are two inputs and one output, the first input is supplied in a competitive market. Suppose there are two solutions θ and $\tilde{\theta}$. To satisfy restrictions imposed by profit maximization, the matrix $\tilde{\Lambda}$ must possess the same structure and properties as Λ.

Because $\Lambda, \tilde{\Lambda}, T$ are invertible, $\Lambda = \tilde{\Lambda}T^{-1}$ implies that

$$T = \Lambda^{-1}\tilde{\Lambda} = \frac{1}{\tilde{\beta}_1\tilde{\phi}_2 + \tilde{\beta}_2 - \tilde{\phi}_2}\begin{bmatrix} \beta_1\tilde{\phi}_2 + \beta_2 - \tilde{\phi}_2 & 0 & 0 \\ \tilde{\beta}_1(\tilde{\phi}_2 - \phi_2) & \tilde{\beta}_2 - \phi_2(1-\tilde{\beta}_1) & \phi_2 - \tilde{\phi}_2 \\ -\beta_1(\tilde{\phi}_1 - \tilde{\beta}_1) - \beta_2\tilde{\beta}_1 + \tilde{\phi}_2\tilde{\beta}_1 & -\beta_1\tilde{\beta}_2 - \beta_2(1-\tilde{\beta}_2) + \beta_1 & \beta_2 - \tilde{\phi}_2(1-\beta_1) \end{bmatrix} \quad \text{(C.1)}$$

Note that $\det(T) = (\beta_1\phi_2 + \beta_2 - \phi_2)/(\tilde{\beta}_1\tilde{\phi}_2 + \tilde{\beta}_2 - \tilde{\phi}_2) \neq 0$ and the solution $\tilde{\theta}$ must have $\tilde{\beta}_1\tilde{\phi}_2 + \tilde{\beta}_2 - \tilde{\phi}_2 < 0$. Thus, the model is not identified unless further restrictions are imposed.

Now consider

$$\Omega = T\tilde{\Omega}T' = \frac{1}{(\tilde{\beta}_1\tilde{\phi}_2 + \tilde{\beta}_2 - \tilde{\phi}_2)^2}\begin{bmatrix} D_{11} & & \\ D_{21} & D_{22} & \\ D_{31} & D_{32} & D_{33} \end{bmatrix},$$

where D_{11}, D_{22}, D_{33} are positive quantities and

$$D_{21} = (\tilde{\beta}_2 + \tilde{\beta}_1\tilde{\phi}_2 - \tilde{\phi}_2)\tilde{\beta}_1(\tilde{\phi}_2 - \phi_2)\tilde{\sigma}_{11},\ D_{31} = (\tilde{\beta}_2 + \tilde{\beta}_1\tilde{\phi}_2 - \tilde{\phi}_2)[-\beta_1(\tilde{\phi}_2 - \tilde{\beta}_2) + \tilde{\beta}_1(\tilde{\phi}_2 - \beta_2)]\tilde{\sigma}_{11},$$

$$D_{32} = (\tilde{\phi}_2 - \phi_2)[\tilde{\beta}_1(\tilde{\phi}_2 - \beta_2) - \beta_1(\tilde{\phi}_2 - \tilde{\beta}_2)]\tilde{\sigma}_{11} + [\tilde{\beta}_2 - \phi_2(1-\tilde{\beta}_1)][\tilde{\beta}_2 - \beta_1\tilde{\beta}_2 - \beta_2(1-\tilde{\beta}_1)]\tilde{\sigma}_{22} +$$
$$+(\tilde{\phi}_2 - \phi_2)(\beta_1\tilde{\phi}_2 + \beta_2 - \tilde{\phi}_2)\tilde{\sigma}_{33}.$$

The restriction that Ω is diagonal implies that $D_{21} = D_{31} = D_{32} = 0$. From D_{21}=0 it follows that $\tilde{\phi}_2 = \phi_2$ since $\tilde{\beta}_1 \neq 0$. Hence, $D_{31} = D_{32} = 0$ implies that

$$\tilde{\beta}_1(\phi_2 - \beta_2) - \beta_1(\phi_2 - \tilde{\beta}_2) = 0 \quad \text{(C.2)}$$

$$\tilde{\beta}_2(1-\beta_1) - \beta_2(1-\tilde{\beta}_1) = 0 \quad \text{(C.3)}$$

The only solution to this system of equations is $\beta_1 = \tilde{\beta}_1$ and $\beta_2 = \tilde{\beta}_2$ implying that T=I and, thus, the model is uniquely globally identified.

For a general model with a productivity shock and n inputs and associated factor prices, the first entry of the first row of T in (C.1) will continue to be non-zero while other entries of the row are zeros. This fixes $\tilde{\phi}_j = \phi_j$ for j=2,…,n and then it is an easy step to show that n-input analogue of (C.2)-(C.3) has unique solution $\tilde{\beta}_j = \beta_j$ for j=1,…,n. This proves part a.

To prove part b, again, without loss of generality, assume that there are two inputs and one output and that the first input is supplied in a competitive market. Suppose there are two solutions θ and $\tilde{\theta}$. Then by assumptions of the proposition, the following matrix must be diagonal

$$\tilde{\Pi} = T^{-1}\Pi T = \frac{1}{|T\|\tilde{\Lambda}|}\begin{bmatrix} D_{11} & 0 & 0 \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix}$$

where D_{11}, D_{22}, D_{33} are non-zero quantities and

$$D_{21} = (\phi_2 - \tilde{\phi}_2)[|\tilde{\Lambda}|\beta_1\Pi_{11} - (\beta_1\tilde{\phi}_2 + \beta_2 - \tilde{\phi}_2)\tilde{\beta}_1\Pi_{22} - (\tilde{\beta}_1(\tilde{\phi}_2 - \beta_2) - \beta_1(\tilde{\phi}_2\text{-}\tilde{\beta}_2))\Pi_{33}],$$

$$D_{31} = |\tilde{\Lambda}|[\beta_1(\phi_2 - \tilde{\beta}_2)\text{-}\tilde{\beta}_1(\phi_2 - \beta_2)]\Pi_{11}\text{-}[\tilde{\beta}_1(1-\beta_2)\text{-}\beta_1(1-\tilde{\beta}_2)]\tilde{\beta}_1(\tilde{\phi}_2 - \phi_2)\Pi_{22} +$$
$$+(\tilde{\beta}_1\phi_2 + \tilde{\beta}_2 - \phi_2)[\tilde{\beta}_1(\tilde{\phi}_2 - \beta_2) - \beta_1(\tilde{\phi}_2 - \tilde{\beta}_2)]\Pi_{33},$$

$$D_{32} = [\tilde{\beta}_2(1-\beta_1) - \beta_2(1-\tilde{\beta}_1)](\tilde{\beta}_1\phi_2 + \tilde{\beta}_2 - \phi_2)(\Pi_{33} - \Pi_{22}),$$

$$D_{23} = (\beta_1\tilde{\phi}_2 + \beta_2 - \tilde{\phi}_2)(\phi_2 - \tilde{\phi}_2)(\Pi_{22} - \Pi_{33}).$$

The restriction that $T^{-1}\Pi T$ is diagonal, implies that $D_{21} = D_{23} = D_{31} = D_{32} = 0$. Suppose that $\Pi_{22} \neq \Pi_{33}$. From D_{23}=0, $(\phi_2 - \tilde{\phi}_2)(\beta_1\tilde{\phi}_2 + \beta_2 - \tilde{\phi}_2) = 0$. Suppose that $\phi_2 = \tilde{\phi}_2$. Then D_{21}=0 and D_{31}=D_{32}=0 imply that

$$\beta_1(\phi_2 - \tilde{\beta}_2) - \tilde{\beta}_1(\phi_2 - \beta_2) = 0 \tag{C.4}$$

$$\tilde{\beta}_2(1-\beta_1) - \beta_2(1-\tilde{\beta}_1) = 0 \tag{C.5}$$

provided that $\tilde{\Pi}_{33} - \tilde{\Pi}_{11} \neq 0$. The only solution to (C.4) and (C.5) is $\beta_1 = \tilde{\beta}_1$ and $\beta_2 = \tilde{\beta}_2$ implying that T=I and, thus, the model is uniquely globally identified almost everywhere.

Now suppose that $\phi_2 \neq \tilde{\phi}_2$ so that $\beta_1\tilde{\phi}_2 + \beta_2 - \tilde{\phi}_2 = 0 \Leftrightarrow \tilde{\phi}_2 = \beta_2/(1-\beta_1)$. Suppose that $\tilde{\beta}_2(1-\beta_1) - \beta_2(1-\tilde{\beta}_1) = 0 \Leftrightarrow \tilde{\beta}_2 = \beta_2(1-\tilde{\beta}_1)/(1-\beta_1)$ from D_{32}=0. Substitute $\tilde{\phi}_2, \tilde{\beta}_2$ in D_{21}=0 and reach the contradiction that $\tilde{\beta}_2 = 0$. Now suppose that $\tilde{\beta}_1\phi_2 + \tilde{\beta}_2 - \phi_2 = 0 \Leftrightarrow \tilde{\beta}_2 = \phi_2(1-\tilde{\beta}_1)$ from D_{32}=0. Substitute $\tilde{\phi}_2, \tilde{\beta}_2$ into $|\tilde{\Lambda}|$ and find that $|\tilde{\Lambda}| = 0$ which contradicts $|T| \neq 0$. Hence, $\phi_2 \neq \tilde{\phi}_2$ leads to contradiction. For a general case with n inputs, one again uses the fact that $\phi_1 = 1$ to fix $\tilde{\phi}_j = \phi_j$ for j=2,…,n and then it is a tedious but straightforward step to show that n-input analogue of (C.4)-(C.5) has unique solution is $\tilde{\beta}_j = \beta_j$ for j=1,…,n almost everywhere. This proves part b. ■

Proof of Proposition 3.
Under assumptions of the proposition, system (8)-(9) can be re-formulated as follows:

$$\boldsymbol{X}_{it} = [\Lambda \mid B]\begin{bmatrix} \boldsymbol{F}_{it} \\ \boldsymbol{M}_{it} \end{bmatrix} + \boldsymbol{\varepsilon}_{it}, \tag{B.6}$$

$$\begin{bmatrix} \boldsymbol{F}_{it} \\ \boldsymbol{M}_{it} \end{bmatrix} = \left[\begin{array}{c|c} \Pi & 0 \\ \hline 0 & R \end{array}\right]\begin{bmatrix} \boldsymbol{F}_{i,t-1} \\ \boldsymbol{M}_{i,t-1} \end{bmatrix} + \begin{bmatrix} \boldsymbol{v}_{it} \\ \boldsymbol{\omega}_{it} \end{bmatrix}, \tag{B.7}$$

where B is the known matrix whose columns are selection vectors e_i (that is e_i is the ith column of matrix I_n) with unity in the row corresponding to the variable with a serially correlated measurement error, $\boldsymbol{M}_{it}$ is a vector of measurement errors, R is a diagonal nonsingular matrix with entries less than unity in absolute value (stationarity of measurement errors), $E(\boldsymbol{\omega}_{it}\boldsymbol{\omega}'_{it}) = \Omega_1$ is a diagonal nonsingular matrix, $E(\boldsymbol{\omega}_{it}\boldsymbol{\omega}'_{js}) = 0$ for all i, j and $s \neq 0$, and $E(\boldsymbol{\varepsilon}_{it}\boldsymbol{\omega}'_{js}) = E(\boldsymbol{\upsilon}_{it}\boldsymbol{\omega}'_{js}) = 0$ for all i, j, s.

To prove global identification, it is sufficient to show that there is no rotation matrix T that preserves the structure of the model. Suppose that such T exits. Then a rotationally equivalent solution must satisfy

$$[\tilde{\Lambda} \mid B] = [\Lambda \mid B]\left[\begin{array}{c|c} T_{11} & T_{12} \\ \hline T_{21} & T_{22} \end{array}\right] = [\Lambda T_{11} + BT_{21} \mid \Lambda T_{12} + BT_{22}].$$

Hence,

$$\tilde{\Lambda} = \Lambda T_{11} + BT_{21} \Leftrightarrow T_{11} = \Lambda^{-1}\tilde{\Lambda} - \Lambda^{-1}BT_{21} \quad \text{(B.8)}$$

$$B = \Lambda T_{12} + BT_{22} \Leftrightarrow T_{12} = \Lambda^{-1}B(I - T_{22}) \quad \text{(B.9)}$$

There are nonlinear restrictions imposed by uncorrelatedness of υ_t and ω_t and block diagonal structure of $\left[\begin{array}{c|c} \Pi & 0 \\ \hline 0 & R \end{array}\right]$. In particular,

$$n \times k \text{ restrictions: } T_{21}\Omega T'_{11} + T_{22}\Omega_1 T'_{12} = 0 \quad \text{(B.10)}$$

$$\tfrac{1}{2}k(k-1) \text{ restrictions: } T_{21}\Omega T'_{21} + T_{22}\Omega_1 T'_{22} \text{ is a diagonal matrix} \quad \text{(B.11)}$$

$$n \times k \text{ restrictions: } \Pi T_{12} - T_{12}T_{22}^{-1}RT_{22} = 0 \quad \text{(B.12)}$$

$$n \times k \text{ restrictions: } T_{21}(T_{11} - T_{12}T_{22}^{-1}T_{21})^{-1}\Pi(T_{11} - T_{12}T_{22}^{-1}T_{21}) = RT_{21} \quad \text{(B.13)}$$

$$1 \text{ restriction: } \det|T_{22}| \neq 0 \quad \text{(B.14)}$$

$$1 \text{ restriction: } \det|T_{11}| \neq 0 \quad \text{(B.15)}$$

Because matrices Λ, Π, B, R have full rank, (B.8)-(B.15) form an overidentified system of quadratic equations. It is easy to verify that $T_{11} = I_n, T_{12} = 0, T_{21} = 0, T_{22} = I_k$ is a solution to the system for any Ω, Ω_1, Π, R. It is straightforward to verify for low dimensional systems (i.e., $n, k \leq 3$) that T=I is the unique real solution. For higher dimensional cases it is hard to verify that T=I is the unique solution. However, since the system is highly overidentified, the measure of alternative solutions that satisfy (B.8)-(B.15) is zero.

Note that restrictions on T_{22}, T_{21} and T_{12} do not pin down the matrix T_{11}. Even if $T_{12} = 0, T_{21} = 0, T_{22} = I_k$, $T_{11} = \Lambda^{-1}\tilde{\Lambda}$ and, therefore, the model is identified if and only if model (8)-(9) is uniquely identified.■

Proof of Proposition 4.

Without loss of generality, consider the system without firm specific effects and measurement error $\boldsymbol{\varepsilon}_{it}$. The residual of the quasi-differenced production function is

$$u_{it} = y_{it} - \hat{\rho}y_{i,t-1} - \hat{b}L_{it} + \hat{\rho}\hat{b}L_{i,t-1} = (\Lambda_2 - \hat{b}\Lambda_1)\boldsymbol{\upsilon}_{it} + (\Lambda_2 - \hat{b}\Lambda_1)(\Pi - \hat{\rho}I)\boldsymbol{F}_{i,t-1},$$

where $\hat{\rho}, \hat{b}$ are "candidate" parameter values. This residual is orthogonal to inputs and output lagged two or more periods if and only if $(\Lambda_2 - \hat{b}\Lambda_1)(\Pi - \hat{\rho}I) = 0$ because $\boldsymbol{F}_{it}$ is serially correlated while $\boldsymbol{\upsilon}_{it}$ is serially uncorrelated.

Note that $\hat{b}$ is a $1 \times (n-1)$ vector and $\hat{\rho}$ is a scalar. Hence, both $\Lambda_2 - \hat{b}\Lambda_1 = 0$ and $\Pi - \hat{\rho}I = 0$ are overidentified because each system has n equations. However, some rows of $\Pi - \hat{\rho}I$ can be non-zero when the corresponding columns of $\Lambda_2 - \hat{b}\Lambda_1$ are equal to zero and vice versa.

Consider first a simple case where the matrix Π is diagonal. If $\hat{\rho}$ is equal to Π_{jj}, one of the diagonal entries of Π, one of the equations in $\Lambda_2 - \hat{b}\Lambda_1 = 0$ can be eliminated, the system becomes just identified and

$\hat{b} = \tilde{\Lambda}_{1j}^{-1}\tilde{\Lambda}_{2j}$ where $\tilde{\Lambda}_{\bullet j}$ is the matrix $\Lambda_{\bullet}$ without the j^{th} column. The Blundell-Bond estimator assumes that the $\hat{\rho}$ is equal to the autocorrelation coefficient for productivity ρ_a so that $\hat{b} = \beta$. However, there are other solutions. For example, the above logic suggests that $\hat{\rho}$ can be equal to the autocorrelation coefficient for wage shocks ρ_w and this choice of $\hat{\rho}$ gives a different solution for $\hat{b}$. It is straightforward to verify that these solutions are locally identified, i.e., the rank of the Jacobian is full:

$$rank\left\{ E\left(\begin{bmatrix} -y_{i,t-1} + bL_{i,t-1} \\ -L_{it} + \rho L_{i,t-1} \end{bmatrix} \boldsymbol{Z}'_{it} \right)\bigg|_{\rho = a_{jj}, b = \tilde{\Lambda}_{1j}^{-1}\tilde{\Lambda}_{2j}} \right\} = rank\left\{ \begin{bmatrix} \Lambda_2 - \tilde{\Lambda}_{1j}^{-1}\tilde{\Lambda}_{2j}\Lambda_1 \\ \Lambda_1\left(\Pi - \Pi_{jj} I\right) \end{bmatrix} E\left(\boldsymbol{F}_{i,t-1}\boldsymbol{Z}'_{it}\right) \right\} = n .$$

It follows that there can be *n* different solutions to $(\Lambda_2 - \hat{b}\Lambda_1)(\Pi - \hat{\rho}I) = 0$ for the case with *n* inputs.

Now suppose that Π is not diagonal. Let $\hat{\rho}$ be equal to an eigenvalue of Π. Then $rank(\Pi - \hat{\rho}I) = n-1$ and, thus, one is back to the case with a diagonal Π, i.e., multiply $\Lambda_2 - \hat{b}\Lambda_1$ by a singular matrix. Hence, for each eigenvalue of Π there is a unique locally-identified solution for $\hat{b}$. Since Π can have *n* distinct eigenvalues (for *n*-1 inputs), there can be *n* solutions for $\hat{b}$.
To prove the last result, note that if $\hat{\rho}$ is equal to a repeated eigenvalue, the rank of $(\Pi - \hat{\rho}I)$ is at most *n*-2. Hence, at least two columns in Λ_1, Λ_2 can be deleted and $\tilde{\Lambda}_2 - \hat{b}\tilde{\Lambda}_1 = 0$ is underidentified so that there are infinitely many solutions for $\hat{b}$. ■

Proof of Proposition 5.
This proof is for the case with multiple inputs which are collected in the vector $\boldsymbol{L}_{it}$. Partition matrix Λ so that follows $\begin{bmatrix} \boldsymbol{L}_{it} \\ \boldsymbol{Y}_{it} \end{bmatrix} = \Lambda \boldsymbol{F}_{it} + \begin{bmatrix} \overline{\boldsymbol{L}}_i \\ \overline{\boldsymbol{Y}}_i \end{bmatrix} + \boldsymbol{\varepsilon}_{it} = \begin{bmatrix} \Lambda_1 \\ \Lambda_2 \end{bmatrix} \begin{bmatrix} \boldsymbol{w}_{it} \\ a_{it} \end{bmatrix} + \begin{bmatrix} \overline{\boldsymbol{L}}_i \\ \overline{\boldsymbol{Y}}_i \end{bmatrix} + \boldsymbol{\varepsilon}_{it}$, where $\overline{\boldsymbol{L}}_i, \overline{\boldsymbol{Y}}_i$ are time invariant effects. For convenience, I define $E(\boldsymbol{F}_{it}\boldsymbol{F}'_{it}) = \Sigma$. It is sufficient to show that the rank of the Jacobian for the moment conditions does not have full rank, i.e., the rank of the Jacobian is smaller than the number of parameters in the model. Define $\vartheta_{it} \equiv \boldsymbol{Y}_{it} - \gamma[\boldsymbol{L}'_{it} \quad \boldsymbol{L}'_{i,t-1} \quad \boldsymbol{Y}'_{i,t-1}]' = \boldsymbol{Y}_{it} - \gamma \boldsymbol{V}_{it}$, which corresponds to the residual from quasi-differenced production function. Apart from having a permanent component, the error term ϑ_{it} has MA(1) structure because of the error term $\boldsymbol{\varepsilon}_{it}$.

Consider the level moments $E(\vartheta_{it}\boldsymbol{Z}'_{it}) = 0$ where $\boldsymbol{Z}_{it} = [\Delta\boldsymbol{L}'_{i,t-2} \quad \ldots \quad \Delta\boldsymbol{L}'_{i,t-k} \quad \Delta\boldsymbol{Y}'_{i,t-2} \quad \ldots \quad \Delta\boldsymbol{Y}'_{i,t-p}]'$. The expected value of the Jacobian of the moment conditions is

$$E\left(-\boldsymbol{V}_{it}\boldsymbol{Z}'_{it}\right) = E\left(\begin{array}{ccc|ccc} \boldsymbol{L}_{it}(\boldsymbol{L}_{i,t-3} - \boldsymbol{L}_{i,t-2})' & \ldots & \boldsymbol{L}_{it}(\boldsymbol{L}_{i,t-k-1} - \boldsymbol{L}_{i,t-k})' & \boldsymbol{L}_{it}(\boldsymbol{Y}_{i,t-3} - \boldsymbol{Y}_{i,t-2})' & \ldots & \boldsymbol{L}_{it}(\boldsymbol{Y}_{i,t-p-1} - \boldsymbol{Y}_{i,t-p})' \\ \hline \boldsymbol{L}_{i,t-1}(\boldsymbol{L}_{i,t-3} - \boldsymbol{L}_{i,t-2})' & \ldots & \boldsymbol{L}_{i,t-1}(\boldsymbol{L}_{i,t-k-1} - \boldsymbol{L}_{i,t-k})' & \boldsymbol{L}_{i,t-1}(\boldsymbol{Y}_{i,t-3} - \boldsymbol{Y}_{i,t-2})' & \ldots & \boldsymbol{L}_{i,t-1}(\boldsymbol{Y}_{i,t-p-1} - \boldsymbol{Y}_{i,t-p})' \\ \hline \boldsymbol{Y}_{i,t-1}(\boldsymbol{L}_{i,t-3} - \boldsymbol{L}_{i,t-2})' & \ldots & \boldsymbol{Y}_{i,t-1}(\boldsymbol{L}_{i,t-k-1} - \boldsymbol{L}_{i,t-k})' & \boldsymbol{Y}_{i,t-1}(\boldsymbol{Y}_{i,t-3} - \boldsymbol{Y}_{i,t-2})' & \ldots & \boldsymbol{Y}_{i,t-1}(\boldsymbol{Y}_{i,t-p-1} - \boldsymbol{Y}_{i,t-p})' \end{array} \right) =$$

$$= -\left[\begin{array}{ccc|ccc} \Lambda_1\Pi^2 D_1 & \ldots & \Lambda_1\Pi^k D_1 & \Lambda_1\Pi^2 D_2 & \ldots & \Lambda_1\Pi^p D_2 \\ \hline \Lambda_1\Pi D_1 & \ldots & \Lambda_1\Pi^{k-1} D_1 & \Lambda_1\Pi D_2 & \ldots & \Lambda_1\Pi^{p-1} D_2 \\ \hline \Lambda_2\Pi D_1 & \ldots & \Lambda_2\Pi^{k-1} D_1 & \Lambda_2\Pi D_2 & \ldots & \Lambda_2\Pi^{p-1} D_2 \end{array} \right] =$$

$$= -\left[\begin{array}{ccc|ccc} \Lambda_1\Pi^2 D_1 & \ldots & \Lambda_1\Pi^k D_1 & \Lambda_1\Pi^2 D_2 & \ldots & \Lambda_1\Pi^p D_2 \\ \hline \Lambda\Pi D_1 & \ldots & \Lambda\Pi^{k-1} D_1 & \Lambda\Pi D_2 & \ldots & \Lambda\Pi^{p-1} D_2 \end{array} \right],$$

where $D_1 = (I - \Pi)\Sigma\Lambda'_1, D_2 = (I - \Pi)\Sigma\Lambda'_2$. Observe that the first row is $\Lambda_1\Pi\Lambda^{-1}$ times the second row; hence, the matrix $E(-\boldsymbol{V}_{it}\boldsymbol{Z}'_{it})$ does not have full rank and parameters of the model are not identified.

Now consider the difference moment conditions $E(\Delta\vartheta_{it}\boldsymbol{Z}'_{it}) = E\{(\Delta\boldsymbol{Y}_{it} - \gamma\Delta\boldsymbol{V}_{it})\boldsymbol{Z}'_{it}\} = 0$ where $\boldsymbol{Z}_{it} = [\boldsymbol{L}'_{i,t-3} \quad \dots \quad \boldsymbol{L}'_{i,t-k} \quad \boldsymbol{Y}'_{i,t-3} \quad \dots \quad \boldsymbol{Y}'_{i,t-p}]'$. Find that the Jacobian is

$$E\left(-\Delta\boldsymbol{V}_{it}\boldsymbol{Z}'_{it}\right) = -E\left(\begin{array}{ccc|ccc} (\boldsymbol{L}_{i,t-1}-\boldsymbol{L}_{i,t})\boldsymbol{L}'_{i,t-3} & \dots & (\boldsymbol{L}_{i,t-1}-\boldsymbol{L}_{i,t})\boldsymbol{L}_{i,t}\boldsymbol{L}'_{i,t-k} & (\boldsymbol{L}_{i,t-1}-\boldsymbol{L}_{i,t})\boldsymbol{Y}'_{i,t-3} & \dots & (\boldsymbol{L}_{i,t-1}-\boldsymbol{L}_{i,t})\boldsymbol{Y}'_{i,t-p} \\ \hline (\boldsymbol{L}_{i,t-2}-\boldsymbol{L}_{i,t-1})\boldsymbol{L}'_{i,t-3} & \dots & (\boldsymbol{L}_{i,t-2}-\boldsymbol{L}_{i,t-1})\boldsymbol{L}'_{i,t-k} & (\boldsymbol{L}_{i,t-2}-\boldsymbol{L}_{i,t-1})\boldsymbol{Y}'_{i,t-3} & \dots & (\boldsymbol{L}_{i,t-2}-\boldsymbol{L}_{i,t-1})\boldsymbol{Y}'_{i,t-p} \\ \hline (\boldsymbol{Y}_{i,t-2}-\boldsymbol{Y}_{i,t-1})\boldsymbol{L}'_{i,t-3} & \dots & (\boldsymbol{Y}_{i,t-2}-\boldsymbol{Y}_{i,t-1})\boldsymbol{L}'_{i,t-k} & (\boldsymbol{Y}_{i,t-2}-\boldsymbol{Y}_{i,t-1})\boldsymbol{Y}'_{i,t-3} & \dots & (\boldsymbol{Y}_{i,t-2}-\boldsymbol{Y}_{i,t-1})\boldsymbol{Y}'_{i,t-p} \end{array}\right) =$$

$$= \left[\begin{array}{ccc|ccc} \Lambda_1\Pi^2 D_1 & \dots & \Lambda_1\Pi^k D_1 & \Lambda_1\Pi^2 D_2 & \dots & \Lambda_1\Pi^p D_2 \\ \hline \Lambda\Pi D_1 & \dots & \Lambda\Pi^{k-1} D_1 & \Lambda\Pi D_2 & \dots & \Lambda\Pi^{p-1} D_2 \end{array}\right].$$

Hence, the difference moment conditions do not have full rank either because the first row is $\Lambda_1\Pi\Lambda^{-1}$ times the second row. The same conclusion follows for the case without measurement errors, i.e., $\boldsymbol{\varepsilon}_{it} = 0$.

Now suppose that there is no firm-specific effect. Then $E\{(\boldsymbol{Y}_{it} - \gamma\boldsymbol{V}_{it})\boldsymbol{Z}'_{it}\} = 0$ with $\boldsymbol{Z}_{it} = [\boldsymbol{L}'_{i,t-2} \quad \dots \quad \boldsymbol{L}'_{i,t-k} \quad \boldsymbol{Y}'_{i,t-2} \quad \dots \quad \boldsymbol{Y}'_{i,t-p}]'$ is a set of valid moment conditions. However, the reduced rank is still a problem as the Jacobian does not have full rank:

$$E\left(\boldsymbol{V}_{it}\boldsymbol{Z}'_{it}\right) = \left[\begin{array}{ccc|ccc} \Lambda_1\Pi^2\Sigma\Lambda'_1 & \dots & \Lambda_1\Pi^k\Sigma\Lambda'_1 & \Lambda_1\Pi^2\Sigma\Lambda'_2 & \dots & \Lambda_1\Pi^p\Sigma\Lambda'_2 \\ \hline \Lambda\Pi\Sigma\Lambda'_1 & \dots & \Lambda\Pi^{k-1}\Sigma\Lambda'_1 & \Lambda\Pi\Sigma\Lambda'_2 & \dots & \Lambda\Pi^{p-1}\Sigma\Lambda'_2 \end{array}\right]$$

where the first row is $\Lambda_1\Pi\Lambda^{-1}$ times the second row.

Now consider $\begin{bmatrix}\boldsymbol{L}_{it}\\ \boldsymbol{Y}_{it}\end{bmatrix} = \Lambda\boldsymbol{F}_{it} + \begin{bmatrix}\bar{\boldsymbol{L}}_i\\ \bar{\boldsymbol{Y}}_i\end{bmatrix} + \begin{bmatrix}B_1\\ B_2\end{bmatrix}\boldsymbol{\upsilon}_{it} + \boldsymbol{\varepsilon}_{it} = \Lambda\boldsymbol{F}_{it} + \begin{bmatrix}\bar{\boldsymbol{L}}_i\\ \bar{\boldsymbol{Y}}_i\end{bmatrix} + B\boldsymbol{\upsilon}_{it} + \boldsymbol{\varepsilon}_{it}$ that nests models where some of the inputs can response contemporaneously to changes in productivity (the matrix *B* is square). This modification also results in level and difference moments not having full rank because the structure of the moment conditions is not changed. For example, consider the difference moment conditions and find that the Jacobian is:

$$E\left(-\Delta\boldsymbol{V}_{it}\boldsymbol{Z}'_{it}\right) = \left[\begin{array}{ccc|ccc} \Lambda_1\Pi^2 D_1 & \dots & \Lambda_1\Pi^k D_1 & \Lambda_1\Pi^2 D_2 & \dots & \Lambda_1\Pi^p D_2 \\ \hline \Lambda\Pi D_1 & \dots & \Lambda\Pi^{k-1} D_1 & \Lambda\Pi D_2 & \dots & \Lambda\Pi^{p-1} D_2 \end{array}\right] +$$
$$+ \left[\begin{array}{ccc|ccc} \Lambda_1\Pi D_3 & \dots & \Lambda_1\Pi^{k-1} D_3 & \Lambda_1\Pi D_4 & \dots & \Lambda_1\Pi^{p-1} D_4 \\ \hline \Lambda D_3 & \dots & \Lambda\Pi^{k-2} D_3 & \Lambda D_4 & \dots & \Lambda\Pi^{p-2} D_4 \end{array}\right],$$

where $\Sigma_\upsilon = E(\boldsymbol{\upsilon}_{it}\boldsymbol{\upsilon}'_{it})$, $D_3 = (I-\Pi)\Sigma_\upsilon B'_1$ and $D_4 = (I-\Pi)\Sigma_\upsilon B'_2$. This matrix does not have full rank because the first row is equal to $\Lambda_1\Pi\Lambda^{-1}$ times the second row. ■

Proof of Proposition 6.

I have shown in Proposition 5 that level and difference moment conditions yield the same Jacobian matrix:

$$D = \begin{bmatrix} \Psi(\tilde{\Gamma}_2 - \tilde{\Gamma}_3) & \Psi(\tilde{\Gamma}_3 - \tilde{\Gamma}_4) & \dots & \Psi(\tilde{\Gamma}_d - \tilde{\Gamma}_{d+1}) \\ \tilde{\Gamma}_1 - \tilde{\Gamma}_2 & \tilde{\Gamma}_2 - \tilde{\Gamma}_3 & \dots & \tilde{\Gamma}_{d-1} - \tilde{\Gamma}_d \end{bmatrix},$$

where $\Psi \equiv [0 \mid I_{p+m-1}]$. Given assumption of the problem, identification of the Blundell-Bond estimator requires that $rank(D) = 2(m+p) - 1$.

Observe that $\tilde{\Gamma}_k - \tilde{\Gamma}_{k+1} = \Upsilon\Pi^k(I-\Pi)\Gamma_0\Upsilon'$. Consider matrix $P = \Psi\begin{bmatrix} 0 & \Pi_{12} & 0 \\ 0 & \Pi_{22} & 0\end{bmatrix}$. Multiply the second row of *D* by *P* and subtract from the first row of *D*. Denote the resulting matrix with D_1:

$$D_1 = \left[\begin{array}{c:c:c:c} \Phi\Pi(I-\Pi)\Gamma_0\Upsilon' & \Phi\Pi^2(I-\Pi)\Gamma_0\Upsilon' & \ldots & \Phi\Pi^{d-1}(I-\Pi)\Gamma_0\Upsilon' \\ \Upsilon\Pi(I-\Pi)\Gamma_0\Upsilon' & \Upsilon\Pi^2(I-\Pi)\Gamma_0\Upsilon' & \ldots & \Upsilon\Pi^{d-1}(I-\Pi)\Gamma_0\Upsilon' \end{array}\right] =$$

$$= \underbrace{\begin{bmatrix} \Phi\Pi \\ \Upsilon\Pi \end{bmatrix}}_{D_1^*} \underbrace{\left[\begin{array}{c:c:c:c} (I-\Pi)\Gamma_0\Upsilon' & \Pi(I-\Pi)\Gamma_0\Upsilon' & \ldots & \Pi^{d-2}(I-\Pi)\Gamma_0\Upsilon' \end{array}\right]}_{D_1^\dagger} = D_1^* D_1^\dagger$$

where $\Phi = \Psi\begin{bmatrix} 0 & 0 & \Pi_{13} \\ 0 & 0 & \Pi_{23} \end{bmatrix}$. The matrices D_1 and D have the same rank. Observe that

$rank(D_1) \le \min\{rank(D_1^*), rank(D_1^\dagger)\}$ and

$$rank(D_1^*) = rank\left(\begin{bmatrix} \Phi\Pi \\ \Upsilon\Pi \end{bmatrix}\right) = rank\left(\begin{bmatrix} \Psi\begin{bmatrix} 0 & 0 & \Pi_{13}\Pi_{33} \\ 0 & 0 & \Pi_{23}\Pi_{33} \end{bmatrix} \\ \begin{bmatrix} 0 & \Pi_{12} & \Pi_{13} \\ 0 & \Pi_{22} & \Pi_{23} \end{bmatrix} \end{bmatrix}\right) \le m+n\,.$$

Hence the model can be identified if $m+n \ge 2(p+m)-1 \Leftrightarrow n \ge 2p+m-1$. ■

Proof of Proposition 7.

Order entries of $\boldsymbol{X}_t$ so that the first element in $\boldsymbol{X}_t$ (and $\boldsymbol{G}_t$) is output y. Without loss of generality suppose that there is only one freely-adjusted input l (labor) such that the first-order condition with respect to this input is $y_t - \phi l_t = w_t$ where ϕ is some constant, $w_t = e_w \boldsymbol{X}_t$ is an exogenous shock, and e_w is the selection vector (i.e., e_w is equal to one at the position of w_t in $\boldsymbol{X}_t$ and zero otherwise). Also, without loss of generality, assume that all other inputs are predetermined. Define Π^y and Π^l as rows of the matrix Π that corresponds to the output y_t and the freely adjusted input l_t. By (A.5),

$$(\Pi^y - \phi\Pi^l)\boldsymbol{X}_{t-1} + (B^y - \phi B^l)\boldsymbol{v}_t = y_t - \phi l_t = w_t = e_w \boldsymbol{X}_t = e_w \Pi \boldsymbol{X}_{t-1} + e_k B \boldsymbol{v}_t\,.$$

Since this holds for any $\boldsymbol{X}_t$ and $\boldsymbol{v}_t$, $(\Pi^y - \phi\Pi^l) = e_w$ and $(B^y - \phi B^l) = e_w$. It follows that $\Pi_{12}^y - \phi\Pi_{12}^l = 0$ and $\Pi_{13}^y - \phi\Pi_{13}^l = e_w \Pi_{33}$. Production function $y_t = \alpha_L l_t + \alpha K_t + a_t$ imposes another restriction on Π: $\Pi_{13}^y - \alpha_L \Pi_{13}^l - e_a \Pi_{33} = \alpha\Pi_{23}$ and $\Pi_{12}^y - \alpha_L \Pi_{12}^l = \alpha\Pi_{22}$. Using these restrictions and the proof of Proposition 6, one finds that the rank of the Jacobian matrix D is:

$$rank(D) \le rank(D_1^*) = rank\left(\begin{bmatrix} \Psi\begin{bmatrix} 0 & 0 & \Pi_{13}\Pi_{33} \\ 0 & 0 & \Pi_{23}\Pi_{33} \end{bmatrix} \\ \begin{bmatrix} 0 & \Pi_{12} & \Pi_{13} \\ 0 & \Pi_{22} & \Pi_{23} \end{bmatrix} \end{bmatrix}\right) = rank\begin{pmatrix} 0 & \Pi_{13}^l\Pi_{33} \\ 0 & \Pi_{23}\Pi_{33} \\ \Pi_{12}^y & \Pi_{13}^y \\ \Pi_{12}^l & \Pi_{13}^l \\ \Pi_{22} & \Pi_{23} \end{pmatrix} = rank\begin{pmatrix} 0 & (\Pi_{13}^l - \frac{1}{\phi-\alpha_L}\alpha\Pi_{23})\Pi_{33} \\ 0 & \Pi_{23}\Pi_{33} \\ 0 & \Pi_{13}^y - \frac{\phi}{\phi-\alpha_L}\alpha\Pi_{23} \\ 0 & \Pi_{13}^l - \frac{1}{\phi-\alpha_L}\alpha\Pi_{23} \\ \Pi_{22} & \Pi_{23} \end{pmatrix}$$

$$= rank\begin{pmatrix} 0 & (\frac{1}{\phi-\alpha_L}e_a - \frac{1}{\phi-\alpha_L}e_w)\Pi_{33}^2 \\ 0 & \Pi_{23}\Pi_{33} \\ 0 & e_w\Pi_{33} \\ 0 & e_a\Pi_{33} \\ \Pi_{22} & \Pi_{23} \end{pmatrix} = rank\begin{pmatrix} 0 & 0 \\ 0 & \Pi_{23}\Pi_{33} \\ 0 & e_w\Pi_{33} \\ 0 & e_a\Pi_{33} \\ \Pi_{22} & \Pi_{23} \end{pmatrix} \le 2(p+m)-2$$

The last equality follows from Π_{33} being diagonal. Since the rank is less than $2(p+m-1)$, the estimator is not identified. ■

Table 1. Profit share s_π as a function of returns to scale in the revenue function and the elasticity of the cost with respect to inputs.

Returns to scale in the revenue function	Elasticity of the cost with respect to inputs		
	$\phi < 1$	$\phi \approx 1$	$\phi > 1$
$\eta < 1$	small s_π	large s_π	large s_π
$\eta \approx 1$	negative s_π	small s_π	large s_π
$\eta > 1$	negative s_π	negative s_π	small s_π

Table 2. Estimates of Returns to Scale: One-output/one-input.

	OLS (1)	SIV (2)	COV (3)	FE (4)	BB (5)
Parameterization A (no measurement error):					
Median bias	0.359	-0.001	0.001	0.272	0.221
Standard deviation	0.003	0.013	0.006	0.003	0.112
Root MSE	0.359	0.013	0.006	0.272	0.251
Parameterization B (i.i.d. measurement error):					
Median bias	0.332	-0.001	0.001	0.217	0.225
Standard deviation	0.004	0.039	0.007	0.005	0.259
Root MSE	0.332	0.039	0.008	0.217	0.348
Parameterization C (serially correlated measurement error):					
Median bias	0.288	-0.267	0.000	0.192	0.145
Standard deviation	0.006	0.024	0.018	0.005	0.200
Root MSE	0.288	0.269	0.018	0.193	0.247
Parameterization D (correlated factor prices and productivity):					
Median bias	0.423	-1.773	0.001	0.313	0.223
Standard deviation	0.004	0.323	0.008	0.005	0.413
Root MSE	0.423	1.844	0.008	0.313	0.475

Note: The table reports median bias, standard deviation and root MSE for OLS, Schmidt's instrumental variables (SIV), covariance (COV), fixed effects (FE), and Blundell-Bond (BB) estimators. The data generating process is (12)-(15): one input and one output. Each experiment is simulated 1,000 times. In all experiments, $\rho_a = 0.9, \rho_w = 0.5, \sigma_{\upsilon a} = \sigma_{\upsilon w} = 1$. In parameterization A, $\rho(\upsilon_{it}^w, \upsilon_{it}^a) = 0, \sigma_{\varepsilon z} = \sigma_{\varepsilon y} = 0$. In parameterization B, $\rho(\upsilon_{it}^w, \upsilon_{it}^a) = 0, \sigma_{\varepsilon z} = \sigma_{\varepsilon y} = 1$. In parameterization C, $\rho(\upsilon_{it}^w, \upsilon_{it}^a) = 0$, $\sigma_{\varepsilon y} = 0$, $\varepsilon_{it}^z = \rho_z \varepsilon_{i,t-e}^z + e_{it}$, $\sigma_e^2 = 1$, $\rho_z = 0.8$. In parameterization D, $\rho(\upsilon_{it}^w, \upsilon_{it}^a) = 0.7, \sigma_{\varepsilon z} = \sigma_{\varepsilon y} = 0$. See text for further details.

Table 3. Estimates of Returns to Scale: One-output/multi-input.

Parameter values		OLS (1)	FE (2)	BB (3)	LP (4)	SIV (5)	COV (6)
Parameterization A (no measurement error):							
β_K	Bias	0.033	-0.011	-0.019	0.001	-0.009	0.000
	St. Dev.	0.004	0.003	0.054	0.003	0.100	0.007
β_L	Bias	0.265	0.270	0.482	0.265	-0.001	-0.001
	St. Dev.	0.006	0.006	0.206	0.006	0.034	0.010
β_M	Bias	0.123	0.096	-0.302	0.087	0.004	0.000
	St. Dev.	0.006	0.006	0.130	0.006	0.049	0.007
η	Bias	0.421	0.356	0.161	0.353	-0.006	0.000
	St. Dev.	0.002	0.003	0.074	0.004	0.054	0.007
	Root MSE	0.421	0.356	0.177	0.353	0.054	0.007
Parameterization B (i.i.d. measurement error):							
β_K	Bias	0.077	0.032	0.042	0.048	-0.017	0.000
	St. Dev.	0.006	0.006	0.230	0.007	0.144	0.011
β_L	Bias	0.269	0.259	0.254	0.269	-0.007	0.001
	St. Dev.	0.007	0.008	0.274	0.007	0.077	0.025
β_M	Bias	0.069	0.042	0.092	0.014	0.000	0.000
	St. Dev.	0.007	0.008	0.264	0.010	0.099	0.038
η	Bias	0.415	0.334	0.388	0.331	-0.024	0.001
	St. Dev.	0.003	0.005	0.260	0.008	0.126	0.025
	Root MSE	0.415	0.334	0.467	0.331	0.129	0.025
Parameterization C (upward sloping labor supply curve):							
β_K	Bias	0.044	-0.002	-0.017	0.010	-0.075	0.000
	St. Dev.	0.004	0.004	0.052	0.003	0.069	0.008
β_L	Bias	0.443	0.446	0.791	0.443	1.810	-0.001
	St. Dev.	0.009	0.009	0.285	0.009	0.120	0.017
β_M	Bias	0.117	0.093	-0.297	0.083	-0.500	0.000
	St. Dev.	0.006	0.006	0.108	0.006	0.026	0.008
η	Bias	0.604	0.536	0.477	0.536	1.235	-0.001
	St. Dev.	0.004	0.005	0.178	0.006	0.045	0.013
	Root MSE	0.604	0.536	0.509	0.536	1.236	0.013

Note: The table reports median bias, st. dev. and MSE of OLS, Schmidt's instrumental variables (SIV), covariance (COV), fixed effects (FE), Blundell-Bond (BB), and Levinsohn-Petrin (LP) estimators. The data generating process is (19)-(22): three inputs and one output. The estimated production function is (18). Each experiment is simulated 1,000 times. In all parameterizations, β_K=0.1η, β_L=0.1η, β_M=0.1η, η=0.55, ρ_r=0.5, ρ_w=0.6, ρ_{p^M} =0.4, ρ_a=0.9, $\sigma_{\upsilon r} = \sigma_{\upsilon w} = \sigma_{\upsilon p^M} = \sigma_{\upsilon a} = 1$. In parameterization A, $\sigma_{\upsilon y}= \sigma_{\varepsilon k}= \sigma_{\varepsilon l}=0$, $\phi = 1$. In parameterization B, $\sigma_{\upsilon y}= \sigma_{\varepsilon k}= \sigma_{\varepsilon l}=1$, $\phi = 1$. In parameterization C, $\sigma_{\upsilon y}= \sigma_{\varepsilon k}= \sigma_{\varepsilon l} =0$, $\phi = 1.5$. See text for further details.

Table 4. Estimates of Returns to Scale: One-output/multi-input with adjustment costs

Parameter values		OLS (1)	FE (2)	BB (3)	LP (4)	SIV (5)	COV (6)
Parameterization A (no measurement error):							
β_K	Bias	0.169	0.250	0.285	0.187	3.714	-0.002
	St. Dev.	0.011	0.025	0.495	0.014	0.512	0.044
β_L	Bias	0.442	0.389	0.345	0.442	-0.093	0.000
	St. Dev.	0.007	0.006	0.170	0.007	0.084	0.018
β_M	Bias	-0.096	-0.087	0.106	-0.186	-0.079	0.000
	St. Dev.	0.005	0.005	0.163	0.007	0.035	0.011
	Bias	0.516	0.552	0.735	0.443	3.542	-0.002
η	St. Dev.	0.009	0.023	0.454	0.011	0.444	0.045
	Root MSE	0.266	0.305	0.747	0.197	12.741	0.002
Parameterization B (i.i.d. measurement error):							
β_K	Bias	0.153	0.059	0.032	0.154	-0.055	0.008
	St. Dev.	0.011	0.014	0.178	0.011	61.911	0.061
β_L	Bias	0.369	0.317	1.105	0.369	2.385	-0.001
	St. Dev.	0.008	0.008	0.339	0.008	51.015	0.034
β_M	Bias	-0.037	-0.047	-0.696	-0.086	-1.893	-0.002
	St. Dev.	0.008	0.007	0.312	0.010	35.505	0.018
	Bias	0.486	0.329	0.441	0.437	0.437	0.005
η	St. Dev.	0.009	0.015	0.230	0.014	46.387	0.057
	Root MSE	0.236	0.108	0.248	0.191	2151.950	0.003
Parameterization C (upward sloping labor supply curve):							
β_K	Bias	-0.014	-0.021	0.339	-0.004	3.026	0.000
	St. Dev.	0.005	0.013	0.454	0.016	0.369	0.046
β_L	Bias	0.455	0.398	0.628	0.455	-0.014	-0.001
	St. Dev.	0.005	0.005	0.228	0.005	0.059	0.028
β_M	Bias	0.039	0.062	-0.140	0.021	-0.012	0.000
	St. Dev.	0.004	0.004	0.074	0.010	0.032	0.011
	Bias	0.481	0.439	0.827	0.472	2.999	-0.001
η	St. Dev.	0.003	0.011	0.442	0.011	0.315	0.048
	Root MSE	0.231	0.193	0.880	0.223	9.096	0.002

Note: The table reports median bias, st. dev. and MSE of OLS, Schmidt's instrumental variables (SIV), covariance (COV), fixed effects (FE), Blundell-Bond (BB), and Levinsohn-Petrin (LP) estimators. The data generating process is (26). The estimated production function is (18). Each experiment is simulated 1,000 times. In all parameterizations, β_K=0.1η, β_L=0.1η, β_M=0.1η, η=0.55, ρ_r=0.5, ρ_w=0.6, ρ_{p^M} =0.4, ρ_a=0.9, $\sigma_{\upsilon r} = \sigma_{\upsilon w} = \sigma_{\upsilon p^M} = \sigma_{\upsilon a} = 1$, $\psi = 6$. In parameterization A, $\sigma_{\upsilon y}$= $\sigma_{\varepsilon k}$= $\sigma_{\varepsilon l}$=0, $\phi = 1$. In parameterization B, $\sigma_{\upsilon y}$= $\sigma_{\varepsilon k}$= $\sigma_{\varepsilon l}$=1, $\phi = 1$. In parameterization C, $\sigma_{\upsilon y}$= $\sigma_{\varepsilon k}$= $\sigma_{\varepsilon l}$ =0, $\phi = 1.5$. See text for further details.

Table 5. Descriptive statistics.

Variable	variation	Mean	Std. Dev.	Min	Max
Ln(real value added)	overall	4.190	1.636	0.269	9.437
	between		1.548	0.793	8.816
	within		0.528	1.790	6.355
Ln(real capital stock)	overall	7.649	1.799	3.432	12.492
	between		1.701	3.935	12.330
	within		0.448	4.973	9.627
Ln(number of employees)	overall	3.763	1.078	2.303	7.145
	between		0.986	2.303	6.709
	within		0.325	1.982	5.095

Note: This table reports descriptive statistics for Chilean manufacturing plants in SIC 3240 industry (Manufacture of footware). The time span is from 1982 to 1996. Value added is nominal value added deflated by the industry price index. Employment includes production and non-production workers. Capital stock, which includes machines and structures, is constructed by perpetual inventory method. See references cited in the text for further information.

Table 6. Covariance and autocovariance matrices.

	Y_t	K_t	L_t
Y_t	261.5	256.3	168.0
K_t	256.3	344.8	176.3
L_t	168.0	176.3	126.0
Y_{t-1}	244.8	251.3	163.7
K_{t-1}	249.2	333.4	171.9
L_{t-1}	163.5	172.5	120.0
Y_{t-2}	239.3	248.0	160.7
K_{t-2}	244.0	324.0	167.8
L_{t-2}	159.5	169.2	116.0
Y_{t-3}	233.3	245.1	157.3
K_{t-3}	239.5	316.1	164.2
L_{t-3}	155.6	166.5	112.3
Y_{t-4}	230.4	243.6	155.3
K_{t-4}	234.1	308.2	159.9
L_{t-4}	152.4	163.5	109.2

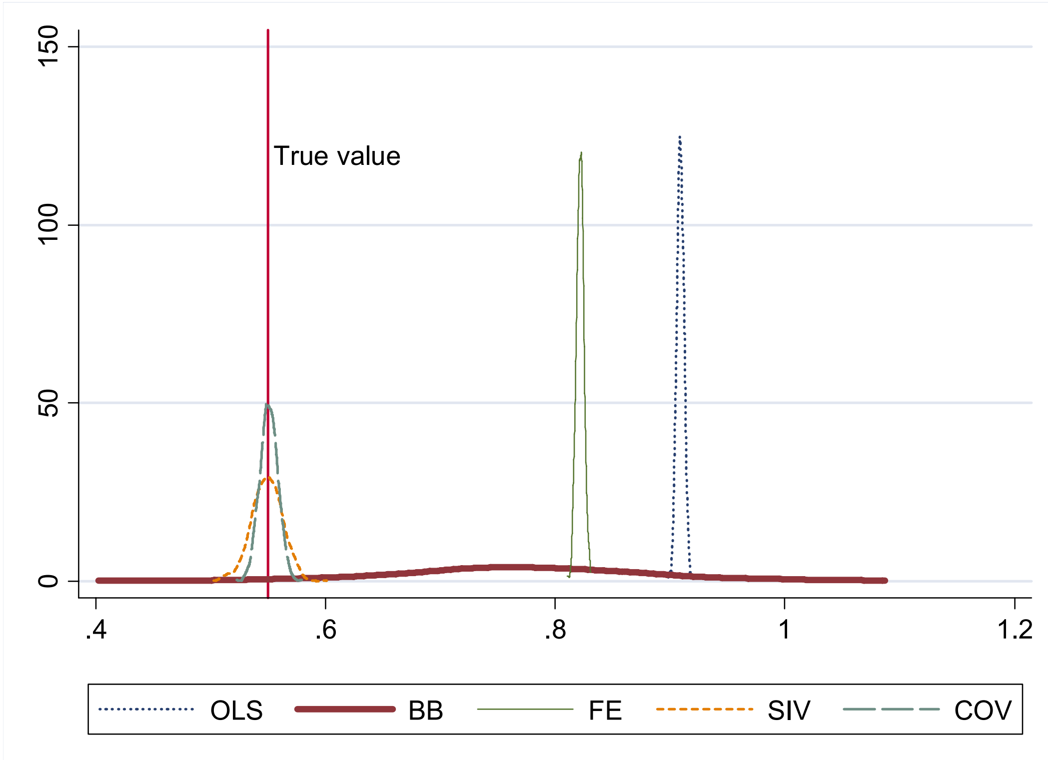
Note: This table presents covariance and autocovariance matrices for logs of value added (Y_t), capital stock (K_t) and labor (L_t) after projecting these variables on the complete set of time dummies. See note to Table 5 for further details.

Table 7. Estimation results.

	COV (1)	OLS (2)	FE (3)	SIV (4)	LP (5)	BB (6)	BB-2 (7)
β_K	0.498	0.198	0.146	-0.398	0.135	0.197	0.2099
	[0.423, 0.514]	(0.017)	(0.029)	(0.050)	(0.054)	(0.130)	(0.123)
β_L	0.697	1.105	0.677	2.952	0.672	0.676	0.6897
	[0.510, 0.730]	(0.029)	(0.047)	(0.131)	(0.073)	(0.132)	(0.128)
ϕ	1.420						
	[1.307, 1.578]						
η	1.172	1.302	0.822	2.554	0.612	0.874	0.899
	[1.008, 1.226]	(0.017)	(0.043)	(0.089)	(0.112)	(0.161)	(0.160)
	Factor prices and technology: standard deviation of innovations and serial correlation						
$\sigma_{\upsilon a}$	0.0306	ρ_{aa}	0.9059	ρ_{aw}	-0.0602	ρ_{ar}	-0.2719
	[0.002, 0.408]		[0.732, 0.961]		[-0.163, 0.031]		[-0.463, -0.042]
$\sigma_{\upsilon w}$	0.0163	ρ_{wa}	0.3118	ρ_{ww}	0.8177	ρ_{wr}	0.0398
	[0.001, 0.442]		[-0.315, 0.533]		[0.074, 0.915]		[-0.090, 0.264]
$\sigma_{\upsilon r}$	0.0657	ρ_{ra}	-0.5024	ρ_{wa}	-0.0359	ρ_{rr}	0.1579
	[0.001, 0.520]		[-0.795, -0.147]		[-0.181, 0.097]		[-0.443, 0.408]
	Measurement errors: standard deviation of innovations and serial correlation						
$\sigma_{\varepsilon v}$	1.862	ρ_v	0.143				
	[1.665, 2.008]		[-0.110, 0.439]				
$\sigma_{\varepsilon k}$	1.8052	ρ_k	0.9305				
	[1.581, 1.972]		[0.903, 0.957]				
σ_{kl}	1.5222	ρ_l	0.7609				
	[1.331, 1.668]		[0.651, 0.844]				

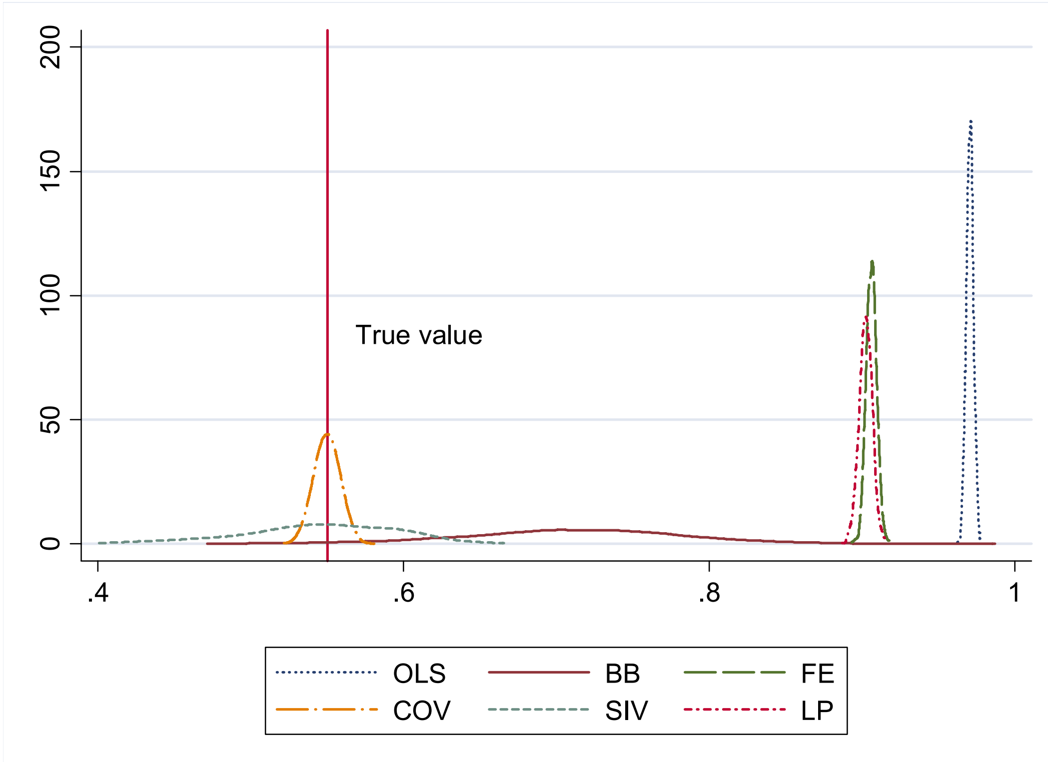
Note: The COV model is described in (27)-(38). 95% bootstrap confidence interval is in square parentheses. FE is fixed effects, LP is Levinsohn-Petrin estimator, BB is Blundell-Bond estimator, SIV is Schmidt's instrumental variables estimator. BB estimator is unrestricted and the reported coefficients are on the current k_{it} and l_{it}. Standard errors are in parentheses. R^2 in OLS regression is 0.92. The LM test does not reject AR(1) model for the error term in the BB estimator.

Figure 1. Kernel density of estimates for returns to scale: BB, OLS, FE, COV, IV estimators



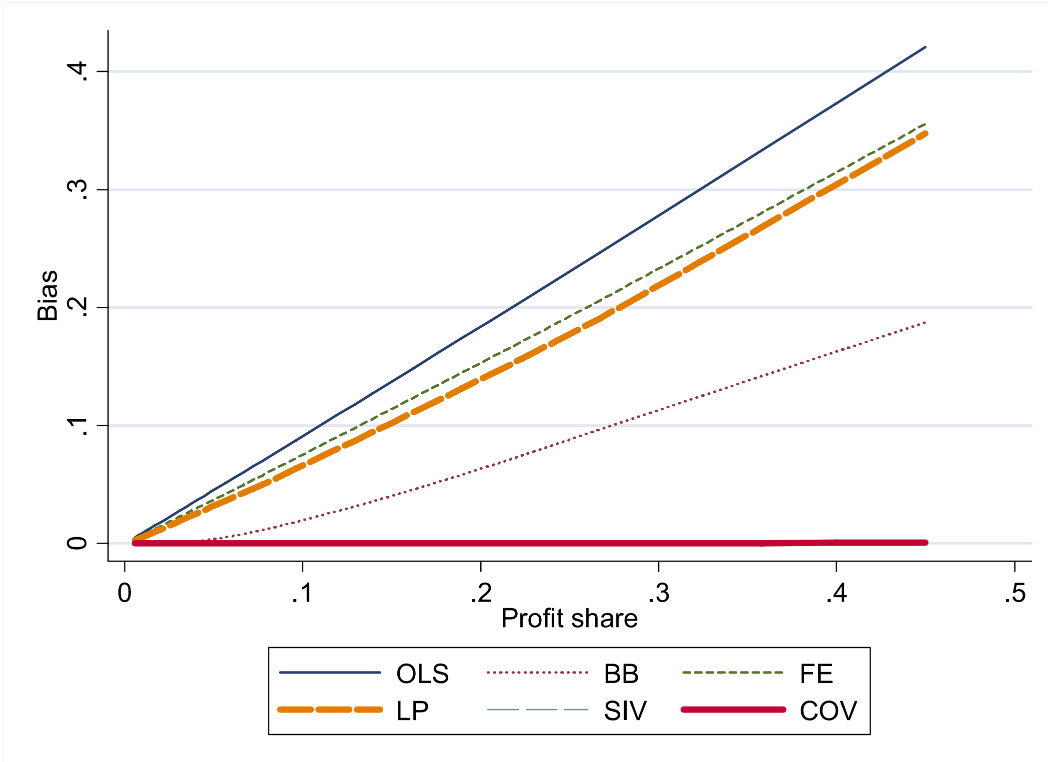
Note: The figure plots kernel densities (Epanechnikov kernel; plug-in optimal bandwidth) of the returns to scale in the revenue function for OLS, Schmidt's instrumental variables (SIV), covariance (COV), fixed effects (FE), and Bond-Blundell (BB) estimators. Parameter values of the data generating process are reported in Panel A, Table 2. Returns to scale are on horizontal axis. The data generating process is (12)-(15). The estimated production function is (16). Each experiment is simulated 1,000 times. See text and Table 2 for further details.

Figure 2. Kernel density of estimates for returns to scale: BB, OLS, FE, LP, COV, IV,



Note: The figure plots kernel densities (Epanechnikov kernel; plug-in optimal bandwidth) of the returns to scale in the revenue function for OLS, Schmidt's instrumental variables (SIV), covariance (COV), fixed effects (FE), and Bond-Blundell (BB), and Levinsohn-Petrin (LP) estimators. Parameter values of the data generating process are reported in Panel A, Table 3. Returns to scale are on horizontal axis. The data generating process is (19)-(22): three inputs and one output. The estimated production function is (18). Each experiment is simulated 1,000 times. See text and Table 3 for further details.

Figure 3. Profit share and bias in returns to scale



Note: The figure reports the bias in the estimated returns to scale in the revenue functions for various values of the profit share. The lines are from lowess which smoothes over 100 replications for each value of the profit share. Parameterization is as in Panel A of Table 3. The estimated production function is (18). BB is Blundell-Bond estimator, FE is fixed effects, SIV is Schmidt's IV, LP is Levinsohn-Petrin estimator, COV is the covariance estimator. SIV essentially coincides with COV in this figure.